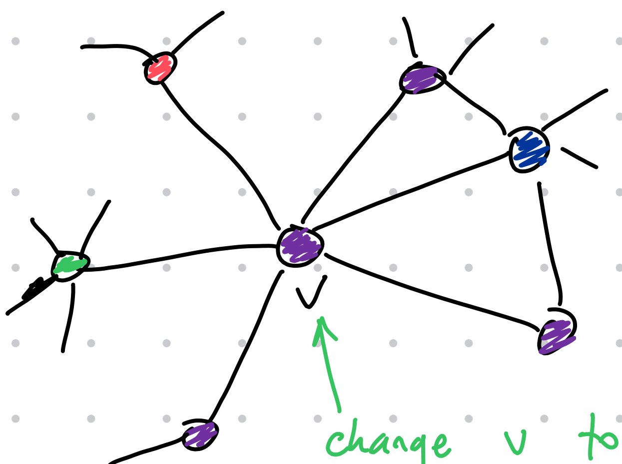
# 5 March 2024

<u>Chromatic number</u> of graph G: the minimum number of colors necessary to properly color the vertices (endpoint of each edge have different colors).

<u>Objective function:</u> count the number of edges having endpoints of the same color

return $(-1)\cdot$ count so that maximum value of zero indicates a proper coloring




change v to green

$$\Delta f = -(\underbrace{-3}_{\text{removing 3 purple pairs}} + 1)$$

negative count

add 1 green pair

plan: $\Delta f = +\left( +\text{num of same-color pairs involving } v \text{ in current coloring} + -\text{num of same-color pairs involving } v \text{ in prop. coloring} \right)$

Absence of a k-clique does not imply that the graph has a proper coloring with fewer than k colors.

I THINK: You can make a graph without any $\underbrace{\text{3-clique}}_{\text{triangle}}$ that has chromatic number a large as you like.

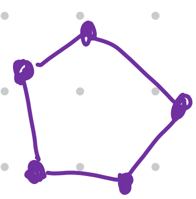
**THEOREM:** If G has max degree k, then G is (k+1)-colorable.

**Proof:** Start coloring the graph, one vertex at a time. When you reach a new vertex, look at its neighbors.

There are at most k neighbors, so there will be one of the k+1 colors available for this vertex.

BROOKS'S THEOREM: If G has max degree k, then G is k-colorable unless G is a $\underbrace{\text{complete graph}}$ or an odd cycle.

k+1 vertices

odd cycle: 

deg. 2

Complexity of Graph Coloring:

n vertices and k colors $\Rightarrow$ $\underbrace{k^n}_{\text{exponential}}$ possible colorings

$\underbrace{k \cdot k \cdot k \cdots k}_{n \text{ vertices}}$

Families of Algorithms / Problems

**CLASS P**: (Decision) problems solvable in polynomial time.

"Polynomial time"

$\underbrace{\text{(Decision) problems}}_{\text{yes/no answer}}$

For input of size n, the algorithm finds a solution in $O(n^k)$ time for some fixed exponent k.

EXAMPLES: selection sort — $O(n^2)$

quicksort — $O(n \cdot \log n)$

Matrix multiplication — multiply 2 $n \times n$ matrices in $O(n^3)$

detecting whether a graph is 2-colorable — $O(n)$ for n vertices

"Nondeterministic Polynomial"

**CLASS NP**: Decision problems verifiable in polynomial time

if someone hands you a solution, you can confirm the solution in polynomial time

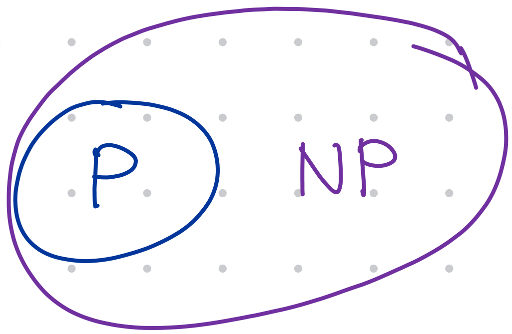
EXAMPLES: Is graph G k-colorable for fixed $k > 2$?

Determining whether a $n \times n$ arrangement is a magic square.

Integer partition problem.

$P \subseteq NP$

[Million dollar question: Is $P = NP$?]


P
NP