29 February 2024

If $f = O(g)$ and $g = O(h)$, then is $f = O(h)$?

STIRLING'S FORMULA:

$$n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$

$$n^2! \sim \sqrt{2\pi n^2}\left(\frac{n^2}{e}\right)^{n^2}$$

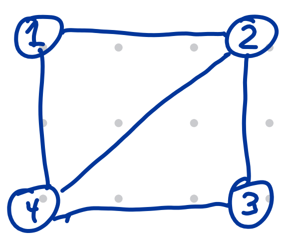
number of square arrangements of $\{1, 2, \ldots, n^2\}$ is $n^2!$

GRAPH THEORY:

A graph is a set of vertices (nodes) and edges connecting pairs of vertices.

EXAMPLE:

$$\begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \left[\begin{array}{cccc} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{array}\right] \end{array}$$



Vertices: $\{1, 2, 3, 4\}$

edges: $\{\{1,2\}, \{2,3\}, \{3,4\}, \{4,1\}, \{2,4\}\}$

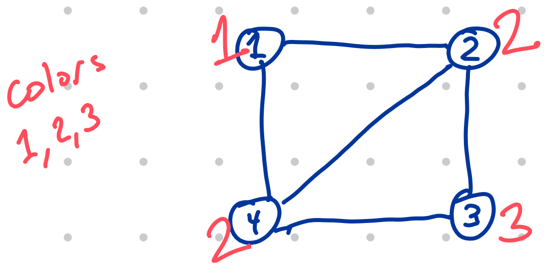
edges are unordered

incidence matrix for a graph with n vertices: an $n \times n$ matrix with a 1 in row i, col j if there is an edge connecting vertices i and j, and 0 otherwise.

For big graphs, incidence matrix uses too much memory and may be sparse (mostly zeros).

For many problems, we want fast access to the list of nodes connected by an edge to any given node.

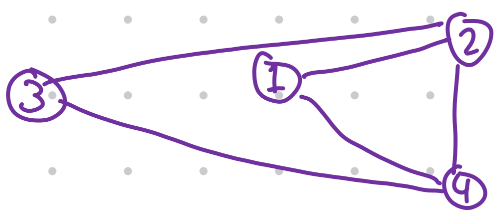
So represent the graph by lists of neighbors for each vertex.



neighbors of 1 → {2,4}; neighbors of 2 → {1,3,4}; neighbors of 3 → {2,4}; neighbors of 4 → {1,2,3}

$\{ \{2,4\}, \{1,3,4\}, \{2,4\}, \{1,2,3\} \}$

↕ Same graph



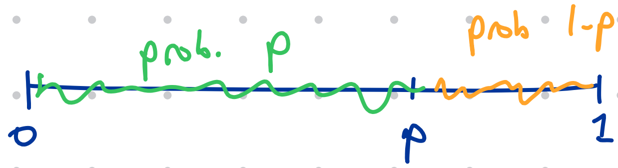
GRAPH COLORING PROBLEM:

Given a graph, what is the minumum number of colors required to color all vertices so that no endpoints of an edge have the same color?

For n vertices and k possible colors, there are

k^n possible colorings.

↑ too many to exhaustively check if k and n are big.



How can we determine whether graph G has a proper coloring (no endpoints of an edge same-colored) with k colors?

→ Simulated Annealing?

What would be required to do this?

- Objective function:
  count the number of "bad" edges, where endpoints have the same color

  Let f( ) = neg of this count, then we want to <u>maximize</u> f.

- Transitions: change the color of a random node.