Homework 4

MATH 348

due at 5pm on Thursday, October 10, 2024

Solve the following problems and communicate your solutions clearly using complete sentences. Your proofs may rely on definitions and theorems stated in the text or given in class.

Remember what the syllabus says about appropriate collaboration, and document what sources you use and what assistance you receive as you work on this homework.

For this homework, you must type your solutions to all of the problems in LaTeX. You may include hand-drawn diagrams in your solutions. Make sure your solutions are easy to read, in order, and clearly labeled. Upload a single file containing your solutions to the Homework 4 assignment on Moodle.

Some of the problems will be graded in detail, and the rest will be graded for completion.

1. (5 points) Let $\mathbb{R}$ have the topology whose open sets are all subsets $U \subset \mathbb{R}$ such that $\mathbb{R} - U$ is either finite or all of $\mathbb{R}$. Show that in this topology every subset of $\mathbb{R}$ is compact.

2. (5 points) Let A be a compact subspace of a Hausdorff space X. Prove that A is closed.

3. (4 points) Exercise 4.10 in the text

4. (6 points) A space is **totally disconnected** if its only connected subspaces are one-point sets. (A subspace is connected if and only if it is connected in the subspace topology.)

Prove or disprove each of the following statements:

(a) If X has the discrete topology, then X is totally disconnected.

(b) If X is totally disconnected, then X has the discrete topology.