

# Disjoint Unions and Product Spaces

MATH 348

1. Describe each of the following spaces

(a)  $\mathbb{R} \amalg \mathbb{R}$

(b)  $\{-1\} \amalg \{+1\}$

(c)  $\mathbb{R} \amalg S^1$

2. Let  $S$  and  $T$  be topological spaces. Prove that  $S \amalg T$  is compact if and only if both  $S$  and  $T$  are both compact.

3. Under what circumstances is  $S \amalg T$  a Hausdorff space? Formulate and prove a statement similar to #2 above.

4. Describe each of the following spaces

(a)  $\mathbb{R} \times S^1$

(b)  $\mathbb{R} \times S^0$

(c)  $S^1 \times [1, 2]$

(d)  $S^1 \times S^1$

5. Suppose  $X$  is a finite set of  $n$  elements and  $Y$  is a finite set of  $m$  elements.

(a) How many elements are there in  $X \amalg Y$ ?

(b) How many elements are there in  $X \times Y$ ?

6. For any product space  $S \times T$ , show that the projection maps  $p_1 : S \amalg T \rightarrow S$  and  $p_2 : S \amalg T \rightarrow T$  are continuous.

7. Let  $S$  and  $T$  be topological spaces. Prove that  $S \times T$  is connected if and only if both  $S$  and  $T$  are both connected.