Homework 4

Math 330

due at 5pm on Thursday, October 5, 2023

Solve the following problems and communicate your solutions clearly. Explain your work using complete sentences, and include diagrams as appropriate.

For this homework, you must type your solutions to all of these problems in LaTeX. Plots/graphs may be drawn by hand or using technology and inserted into your LaTeX document. Make sure your solutions are easy to read, in order, and clearly labeled. Upload a single file containing your solutions to the Homework 4 assignment on Moodle.

The numbered exercises below are from the textbook. Assume that Fourier series are 2π-periodic (that is, computed on the interval $[-\pi, \pi]$ unless otherwise noted.

1. (9 points) Exercise 3.2.1 (c), (d), and (g)

For each function, produce a properly labeled graph that shows the function together with one or more partial sums of its Fourier series on the interval $[-3\pi, 3\pi]$.

2. (6 points) Exercise 3.2.2 (a) and (b)

For each function, produce a properly labeled graph that shows the function together with one or more partial sums of its Fourier series on the interval $[-3\pi, 3\pi]$.

3. (2 points) Exercise 3.2.3

Recall that $\sin^2 x = \dfrac{1-\cos(2x)}{2}$ and $\cos^2 x = \dfrac{1+\cos(2x)}{2}$.

4. (4 points) Exercise 3.2.6 (a), (c), (d), and (e)

Note that $\sinh x = \frac{e^x - e^{-x}}{2}$ is the hyperbolic sine function.

5. (3 points) Exercise 3.2.18

6. (6 points) Consider the differential equation

$$\frac{d^2 f}{dx^2} + \lambda f = 0.$$

Determine the eigenvalues λ and corresponding eigenfunctions if f satisfies the following boundary conditions. Analyze the three cases $\lambda > 0$, $\lambda = 0$, and $\lambda < 0$. You may assume the eigenvalues are real.

(a) $f(0) = 0$ and $f(1) = 0$

(b) $\frac{df}{dx}(0) = 0$ and $f(1) = 0$

(c) $f(0) = 0$ and $\frac{df}{dx}(1) + f(1) = 0$ (Here it might not be possible to obtain exact expressions for all of the eigenvalues. Just get as far as you are able.)