Derivative Approximations

Math 330

1. Let u be a C^3 function.

(a) Write the Taylor polynomial of degree 2 for $u(x+h)$.

(b) Write the Taylor polynomial of degree 2 for $u(x-h)$.

(c) Subtract one Taylor polynomial from the other, and solve for $u'(x)$. You now have an approximation for the first derivative. What is the order of its truncation error?

2. Let u be a C^4 function.

(a) Write the Taylor polynomial of degree 3 for $u(x+h)$.

(b) Write the Taylor polynomial of degree 3 for $u(x-h)$.

(c) Add the two Taylor polynomials and solve for $u''(x)$. You now have an approximation for the second derivative. What is the order of its truncation error?

3. Here is one more example that suggests a more general approach for finding finite difference approximations to derivatives. We seek an approximation of $u'(x)$ using the values $u(x-h)$, $u(x)$, $u(x+h)$, and $u(x+2h)$.

(a) Write the Taylor polynomials of degree 3 for $u(x-h)$, $u(x)$, $u(x+h)$, and $u(x+2h)$.

(b) View your Taylor polynomials as a system of linear equations of the following form:

$$\begin{bmatrix} u(x-h) \\ u(x) \\ u(x+h) \\ u(x+2h) \end{bmatrix} = A \begin{bmatrix} u(x) \\ u'(x)h \\ u''(x)h^2 \\ u'''(x)h^3 \end{bmatrix}$$

where A is a 4×4 coefficient matrix. What is this matrix?

(c) Find A^{-1}. (Use technology.) Use the entries in the second row of A^{-1} to write down an approximation of $u'(x)$. What is the order of the truncation error?

(d) Note that the entries of A^{-1} also give you approximations of $u''(x)$ and $u'''(x)$.