Fourier Series

Math 330

1. Match the following $2\pi-$periodic functions with their Fourier series *without* solving for the coefficients.

(a) $f(x) = x^2$ for $x \in [-\pi, \pi]$

I. $f(x) = \dfrac{8}{\pi} \displaystyle\sum_{k=1}^{\infty} \frac{k(-1)^k}{1-4k^2} \sin(kx)$

(b) $f(x) = x(\pi^2 - x^2)$ for $x \in [-\pi, \pi]$

II. $f(x) = \dfrac{\pi^2}{3} + 4 \displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{k^2} \cos(kx)$

(c) $f(x) = \sin\left(\dfrac{x}{2}\right)$ for $x \in [-\pi, \pi]$

III. $f(x) = -12 \displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{k^3} \sin(kx)$

(d) $f(x) = \begin{cases} \frac{\pi}{2} + x, & -\pi \leq x < 0 \\ \frac{\pi}{2} - x, & 0 \leq x \leq \pi \end{cases}$

IV. $f(x) = \dfrac{2}{\pi} \displaystyle\sum_{k=1}^{\infty} \frac{1-(-1)^k}{k^2} \cos(kx)$

2. Let $f(x) = 1 - x$ be defined on $x \in [0, 1]$.

(a) Sketch $\tilde{f}(x)$, the odd 2-periodic extension of $f(x)$. That is, $\tilde{f}(x)$ should be an odd function with period 2.

(b) If you were to write a trigonometric series that converges $\tilde{f}(x)$, what terms would you put in this series?

(c) Compute the coefficients for your trigonometric series from part (b). Then plot some partial sums of this series together with $\tilde{f}(x)$.

(d) Sketch $\hat{f}(x)$, the even 2-periodic extension of $f(x)$. That is, $\hat{f}(x)$ should be an even function with period 2.

(e) If you were to write a trigonometric series that converges $\hat{f}(x)$, what terms would you put in this series?

(f) Compute the coefficients for your trigonometric series from part (e). Then plot some partial sums of this series together with $\hat{f}(x)$.