Orthogonality

Math 330

1. Let m and n be integers. Prove the identity

$$\int_{=\pi}^{\pi} \sin(mx)\sin(nx)\ dx = \begin{cases} 0, \text{if } m \neq n, \\ \pi, \text{if } m = n \neq 0. \end{cases}$$

2. We would like to be able to write "any" function $f(x)$ as a sum of sine functions. Assume that $f(x)$ can be written as

$$f(x) = \sum_{k=1}^{\infty} b_k \sin(kx).$$

Multiply both sides of the equation above by $\sin(mx)$, then integrate both sides from $-\pi$ to π with respect to x. Interchange integration and summation and solve for b_k.

3. Use your newfound power to write $f(x) = x$ as a sum of sine functions. Write out the first 6 terms in the series. Use technology to plot your series.

4. Derive an identity for cosine of the following form:

$$\int_{-\pi}^{\pi} \cos(mx)\cos(nx)\ dx = \begin{cases} & n \neq m \\ & n = m \neq 0 \\ & n = m = 0 \end{cases}$$

5. Can you write $f(x) = x$ as a sum of cosine functions? Why or why not?