

6. Consider the nonuniform transport equation

$$\frac{\partial u}{\partial t} + x \frac{\partial u}{\partial x} = 0.$$

(a) Sketch some slope lines tangent to the characteristic curves for this equation. What is the shape of the characteristic curves?

(b) The characteristic curves are given by what functions $x(t)$?

(c) Suppose $u(t, x)$ satisfies this differential equation. Describe in words how the graph of $u(t, x)$ changes as t increases. Optionally, you may assume an initial condition such as $u(0, x) = e^{-x^2}$.

(d) Give an expression for the solution $u(t, x)$.