Final Project Information

Math 330

The final project is your opportunity to learn about a topic that we didn't have the chance to cover during the semester—and to learn about many other topics from your classmates! Working in a team of 3 or 4 students, you will discuss the material as you figure it out, which leads to a deeper understanding of the material and a higher quality product.

You will submit a paper with your findings and deliver a short presentation on your work. Your research is not expected to be original or novel, however, it is expected that you do a thorough literature review, discussing what has been done by others (clearly citing that work) and presenting some analysis of your study. Your project should include as much of the following as is reasonable:

- Clear statement of your research goals and description of the PDE or method that you have studied.
- Thorough literature review. Why is this topic important, what is its history, and what are the important publications about it? (A comprehensive bibliography is expected, not just the sources you happened to consult, but a complete set of sources you would recommend that a reader consult to learn a complete picture.)
- Discussion of the tools and methods used to analyze the PDE and what they determine.
- Discussion of the applications of the PDE or method that you studied.
- Discussion of the importance of the PDE or method for the scientific community and what current work is being done.

Paper

Your paper must be typed in LaTeX and submitted on Moodle by start of the scheduled final exam period (Wednesday, December 18, 9:00am).

The title of your paper should be brief and should describe the content of your paper. An abstract not exceeding 200 words that summarizes the principal techniques and conclusions of your work must appear following your title, but before the content of your paper. The recommended length of your paper is 10–15 pages, though lengths vary depending on formatting, inclusion of diagrams, etc.

For some guidelines about writing a math paper, consult *Guidelines for Good Mathematical Writing* by Francis Su[1] and *How to Write Mathematical Papers* by Bruce Berndt.[2]

All illustrations must be of professional quality with no handwritten elements. Illustrations must be numbered consecutively and cited in the text. Please make sure all figures are legible with large enough font and clearly labeled with a descriptive caption.

Presentation

During the scheduled final exam period, your group will give a 10-15 minute presentation. Your talk should summarize the analysis you have done in your paper. While the presentation does not have to be as detailed or as technical as your paper, it should give your audience a clear idea of what you have done and what you have found.

Your talk should involve a few slides, prepared using the technology of your choice. Make sure your slides are legible, with figures clearly labeled. Slides with pictures and concise text tend to be more informative than those filled with equations. Include references in your slides as appropriate.

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[1] Francis Edward Su, *Guidelines for Good Mathematical Writing*, `https://www.math.hmc.edu/~su/math131/good-math-writing.pdf`

[2] Bruce Berndt, *How to Write Mathematical Papers*, `https://faculty.math.illinois.edu/~berndt/writingmath.pdf`.

Timeline

The following is a schedule for work on the projects.

- **Early November:** Think about what you might want to research for your project. Identify three possible topics.
- **Tuesday, November 12:** Complete the Project Planning Survey on Moodle. This will ask you for possible topics and who you do (or don't) want to work with.
- **Tuesday, November 19:** Projects and teams finalized.
- **Thursday, December 5:** Outline of paper due.
- **Tuesday, December 10:** Draft of paper due.
- **Wednesday, December 18:** Paper due; presentations given during final exam time.

Ideas

Some possible topics for the final project appear below. This list is not intended to be exhaustive—feel free to come up with other ideas as well!

1. **Finite Element Methods:** The Finite Difference Method (FDM) is the oldest method for approximating solutions to PDEs. The FDM uses a topologically square network of lines to construct the discretization of the PDE. This is a potential bottleneck of the method when handling complex geometries in multiple dimensions. This issue motivated the use of an integral form of the PDEs and subsequently the development of the finite element techniques. The use of integral formulations is advantageous as it provides a more natural treatment of Neumann boundary conditions as well as that of discontinuous source terms due to their reduced requirements on the regularity or smoothness of the solution. Moreover, they are better suited than the FDM to deal with complex geometries in multi-dimensional problems as the integral formulations do not rely in any special mesh structure.
2. **Pattern Formation:** Explore possible spatial pattern formation with reaction-diffusion systems. It is fun to see patterns emerge (e.g., patterns on animal coats) and to work towards understanding of these phenomena (e.g., morphogenesis underlies similar processes). This field began with Turings ideas from 1952. He found that it is possible for chemicals (or morphogens) to react and diffuse in such a way that the steady state displays heterogeneous spatial patterns of the concentrations of these chemicals.
3. **Traffic Flow:** There are many fascinating mathematical aspects to traffic flow. Possible topics include traffic jam formation, evacuation routes, and flow through intersections. Many techniques, both elementary and sophisticated, have been used to study traffic flow mathematically in recent years.
4. **Vortex Filament Flow, Hasimoto Transform, and the Cubic Nonlinear Schrödinger Equation:** The vortex filament flow is an equation modeling the evolution of a vortex filament under its own vorticity. Hasimoto discovered that this equation could be transformed to the cubic nonlinear Schrödinger equation, which is known to possess some remarkable properties such as infinite hierarchies of symmetries and conservation laws, bi-Hamiltonian structure, and a host of others.
5. **The Cable Equation: Signal Propagation along a Neuron:** Cable theory has its roots in studying transmissions along underwater telegraph lines. Later, these ideas were applied to electric currents in neurons. The partial differential equation known as the cable equation is used today to model the propagation and interaction of electrical signals in spatially extended nerve cells.

6. **Wave Guides and Fiber Optics:** A wave guide is just that, a method to guide waves. Wave guides confine energy to one or two dimensions to ensure the waves propagate efficiently because the energy of the waves decays quickly in three dimensions. There are many types of waves, and similarly there are many different wave guides (e.g., acoustic waves or electromagnetic waves in a cylindrical wave guide). Fiber optics are physically different than other wave guides because light travels in a solid medium (as opposed to air) which is encased in a second material.

7. **Nonlinear/Dispersive Waves and Analysis of Solitons:** Solitons are special solutions to time-dependent nonlinear partial differential equations. Over time, even under natural disturbances and noise, they preserve their shape and speed of travel. They can represent water waves or pulses of light in a fiber optic cable or a laser. They have many important and fascinating applications. For example, you might study various solutions to the Korteweg-deVries equation (which represents water waves), such as multiple-soliton collisions, or collisions between solitons and other objects. You might also be interested in studying the soliton solutions for different types of equations and comparing their properties.

8. **Euler Equations and Navier-Stokes Equations:** Euler equations and Navier-Stokes equations are systems of PDEs that describe fluid flow. Euler equations derive from a conservation law and a balance of momentum and energy, and are seen as a non-viscous version of the Navier-Stokes equations. The Navier-Stokes equations have a million dollar prize attached to them for proofs about the existence and smoothness of solutions.

9. **Black-Scholes equation:** The Black-Scholes equation is a PDE that models the evolution of call prices. It is widely used in the modeling of financial markets. It is related to, but fundamentally different from, the diffusion equation (heat equation).

10. **Green's Functions:** Green's functions are used to solve PDEs over certain types of domains with symmetry. Often one can find a "fundamental solution" by which all other solutions can be constructed using the given boundary conditions and initial data.

11. **Nonlinear Diffusion, Burgers' Equation, and The Hopf-Cole Transformation:** Burgers' equation is one of the simplest nonlinear PDEs, and is a simple model of a nonlinear "heat" equation. The Hopf-Cole transformation is a remarkable change of variables which transforms Burgers' equation into the heat equation. The inviscid Burgers' equation produces shock-wave solutions, but typically the viscosity parameter smooths traveling wave solutions.

12. **Modeling the Spread of Epidemics: An Age-Structured Model:** Traditional models for the spread of infectious diseases are based on systems of ordinary differential equations (e.g., the classic SIR model). However, the risk for contracting some diseases depends on age. Therefore, it is necessary to use an age-structured model, which results in a system of PDEs.

13. **Radiative Transfer Equation:** This equation describes how radiation propagates through a medium by absorption, emission, and scattering. There are various solution techniques, such as the Eddington approximation and the discrete ordinates method.

14. **Pick your own topic!** PDEs are everywhere—they arise in economics, engineering, biology, physics, chemistry, and many other fields.