Homework 1

Math 330

Solve the following problems. For this first homework, you must type (in LaTeX) your solutions to at least *two* of these problems. (You may handwrite or type the remaining solutions.)

Submit your work either on Moodle or in the homework mailbox (RMS level 3, near the fireplace) by 4:00pm on **Thursday, September 12**.

1. Solve the initial-value problem: $\dfrac{dy}{dt} = 3y + 2e^{3t}$ and $y(0) = 2$.

2. Solve each of the following initial-value problems. Describe the behavior of the solution $y(t)$ as $t \to \infty$ and $t \to -\infty$.

(a) $y'' = 0, \quad y(0) = 3,\ y'(0) = 2$

(b) $y'' + 2y' + y = 0, \quad y(0) = 1,\ y'(0) = -1$

(c) $y'' - y' - 6y = 0, \quad y(0) = 1,\ y'(0) = 2$

3. Solve each of the following boundary-value problems.

(a) $y'' + 2y' + 2y = 0, \quad y(0) = 0,\ y\left(\frac{\pi}{2}\right) = 2$

(b) $y'' + 2y' + 2y = 0, \quad y(0) = 0,\ y(\pi) = 0$

4. Problem 1.2.1 (a) and (b) from the textbook. (Explain in your own words.)

5. Problem 1.2.8 from the textbook. (You may assume that $c(x)$, $\rho(x)$, and $A(x)$ are also known.)

6. Suppose $u(x,t)$ is the concentration of particles in a thin tube of length L. The flux of particles at (x,t) is given by $\phi(x,t)$ and the rate of creation/degradation of particles is given by $\sigma(x,t)$ (concentration per time). Derive the integral and differential forms of the conservation equation for a situation where the cross-sectional area varies both spatially and temporally ($A = A(x,t)$). Note that your equation will contain ϕ since we are not given a relationship between ϕ and u.

Hint: For the heat energy example in Section 1.2 of the textbook, the integral conservation equation is Equation (1.2.4), and the differential form is Equation (1.2.5). In general, the conservation equation has the form:

rate of change in the quantity of particles	=	quantity of particles flowing across boundaries per unit time	±	rate of creation or degradation of particles