Homework 7

Math 330

Type (in LaTeX) your solutions to the following problems. Submit them either on Moodle or in the homework mailbox (RMS level 3, near the fireplace) by 4:00pm on **Thursday, November 1**.

1. Problem 4.4.3

2. Problem 4.4.6 — You should start with the solution to the wave equation given in equation (4.4.11) and show that it can be written in the form $u(x,t) = R(x-ct) + S(x+ct)$. *Hint*: use $\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$ and a similar identity for $\sin\alpha\sin\beta$.

3. Problem 4.4.7

For the remaining problems, consider the wave equation on an infinite string:

$$\begin{cases} \frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} & -\infty < x < \infty, \quad t > 0 \\ u(x,0) = f(x) & -\infty < x < \infty \\ u_t(x,0) = g(x) & -\infty < x < \infty \end{cases} \tag{$*$}$$

4. Use D'Alembert's solution (refer to the worksheet/notes from class) to solve the wave equation $(*)$ with $c = 1$,

$$f(x) = \begin{cases} x+1, & -1 \le x \le 0 \\ 1-x, & 0 < x \le 1 \\ 0 & \text{otherwise,} \end{cases}$$

and $g(x) = 0$. Sketch the solution for each of the following values of t: 0, $\frac{1}{2}$, 1, and $\frac{3}{2}$. Interpret your solution in terms of traveling waves.

5. Use D'Alembert's solution to solve the wave equation $(*)$ with $c = 1$, $f(x) = 0$, and $g(x) = \sin(x)$. Sketch the solution for each of the following values of t: 0, $\frac{\pi}{4}$, $\frac{\pi}{2}$, $\frac{5\pi}{4}$. Interpret your solution in terms of traveling waves.

6. Use D'Alembert's solution to solve the wave equation $(*)$ with $c = 1$, $f(x) = 0$, and $g(x) = e^{-x^2}$. Sketch the solution for each of the following values of t: $0, 1, 3, 5$. Interpret your solution in terms of traveling waves.

7. What happens if you change the value of c in the wave equation? Choose one of problems 4, 5, or 6, and experiment with different values of c. How does your choice of c affect the graphs of the solution? What is the role of c in the wave equation?