Homework 1

Math 330

Type (in LaTeX) your solutions to the following problems. Submit them either on Moodle or in the homework mailbox (RMS level 3, near the fireplace) by 4:00pm on **Thursday, September 14**.

1. Suppose $u(x,t)$ is the concentration of particles in a thin tube of length L. The flux of particles at (x,t) is given by $\phi(x,t)$ and the rate of creation/degradation of particles is given by $\sigma(x,t)$ (concentration per time). Derive the integral and differential forms of the conservation equation for a situation where the cross-sectional area varies both spatially and temporally ($A = A(x,t)$). Note that your equation will contain ϕ since we are not given a relationship between ϕ and u.

Hint: For the heat energy example in Section 1.2 of the textbook, the integral conservation equation is Equation (1.2.4), and the differential form is Equation (1.2.5). In general, the conservation equation has the form:

rate of change in the quantity of particles	=	quantity of particles flowing across boundaries per unit time	±	rate of creation or degradation of particles

2. *A few review problems:* Solve each of the following initial-value problems. Describe the behavior of the solution $y(t)$ as $t \to \infty$ and $t \to -\infty$.

(a) $y'' = 0, \quad y(0) = 3,\ y'(0) = 2$

(b) $y'' + 2y' + y = 0, \quad y(0) = 1,\ y'(0) = -1$

(c) $y'' - y' - 6y = 0, \quad y(0) = 1,\ y'(0) = 2$

3. Problem 1.3.2 from the textbook.

4. Problem 1.4.1(cdfg) from the textbook.

5. Problem 1.4.2 from the textbook.

6. Problem 1.4.3 from the textbook. Include a plot of your solution.

7. Problem 1.4.6(a) from the textbook.