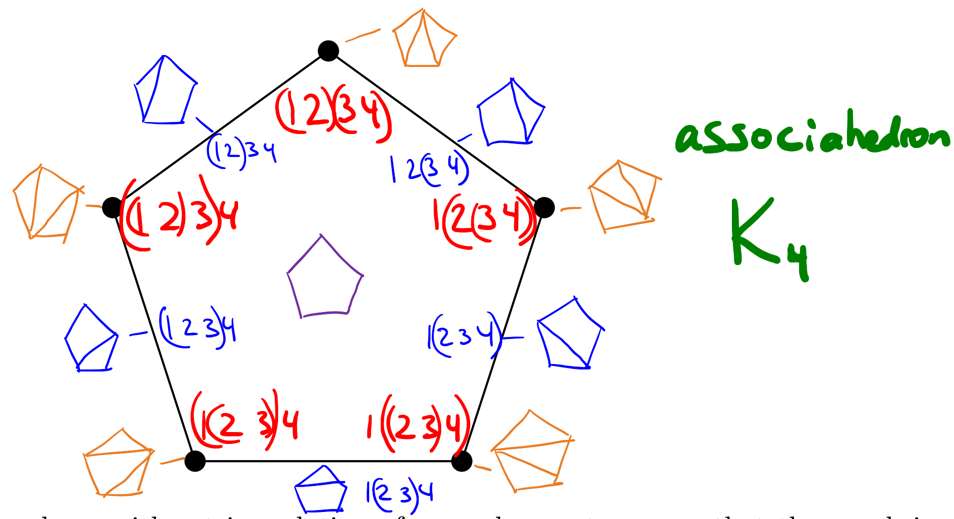
Particle Collisions

Math 282 Computational Geometry

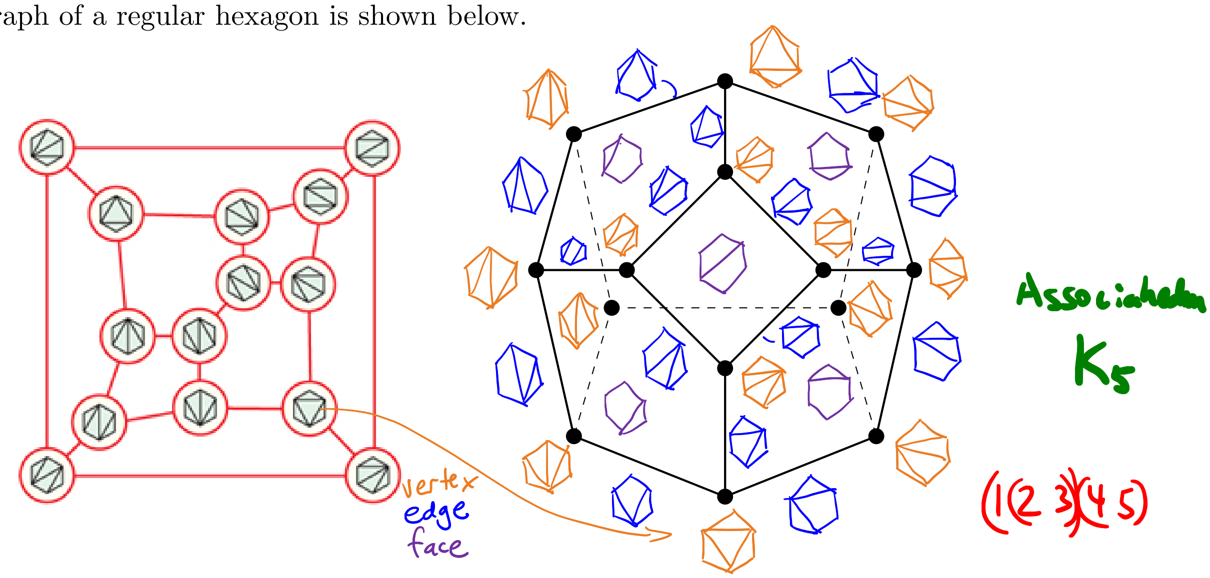
1. Recall that the flip graph of a regular pentagon can be drawn as a pentagon.



(a) Label each vertex above with a triangulation of a regular pentagon, so that the graph is the flip graph of a regular pentagon.

(b) Each edge in the flip graph connects two vertices are labeled with triangulations that have exactly one common diagonal. Label each edge to show its associated diagonal in a regular pentagon.

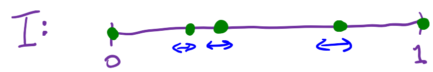
2. The flip graph of a regular hexagon is shown below.



(a) Label each vertex of the following polyhedron with a triangulation of the hexagon such that each edge of the polyhedron corresponds to an edge flip. This shows that the vertices and edges of the polyhedron have the same structure as flip graph!

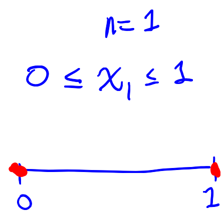
(b) How could you label the edges and faces of the polyhedron consistently with the vertex labels?

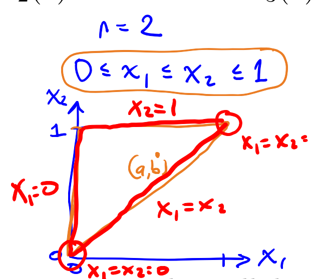
The labeled pentagon is known as the *associahedron* K_4, and the labeled polyhedron is known as the *associahedron* K_5.

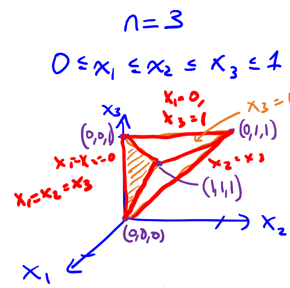


We now consider the configuration space of particles moving along the unit interval $I = [0, 1]$. Suppose there are fixed particles at the endpoints 0 and 1, and n particles in the interior of the interval. Particles are indistinguishable and are able to touch adjacent particles. Let $C_n(I)$ be this configuration space.

3. How would you describe the configuration space $C_1(I)$ for one particle in the interior of the interval? Make a sketch. How about $C_2(I)$? How about $C_3(I)$?

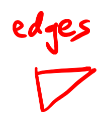






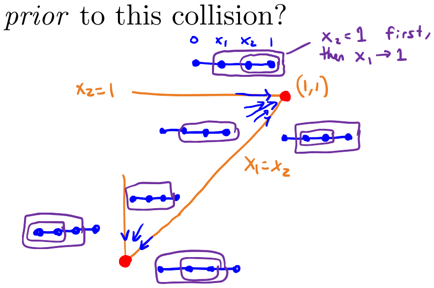
4. Some configurations result in two or more particles colliding. How are these configurations represented each configuration space $C_1(I)$, $C_2(I)$, and $C_3(I)$?

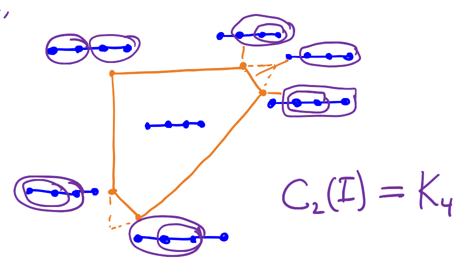
endpoints



edges and faces

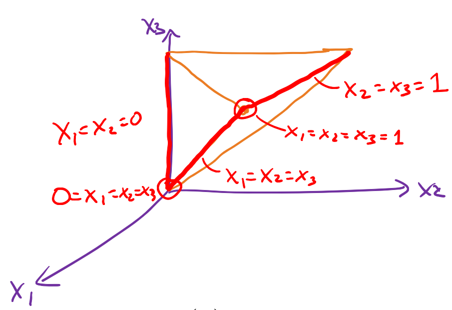
5. We now consider *ways* in which three or more particles can collide. For example, in $C_2(I)$, suppose that both mobile particles have collide with the endpoint 1. What are the possible configurations just *prior* to this collision?





6. By carefully examining the collisions at 0 and 1, how is the configuration space $C_2(I)$ related to the associahedron K_4?

7. What are collisions of three or more particles represented in the configuration space $C_3(I)$? How could you modify this configuration space to indicate all of the *ways* in which these collisions can occur?



8. How is the configuration space $C_3(I)$ related to the associahedron K_5?