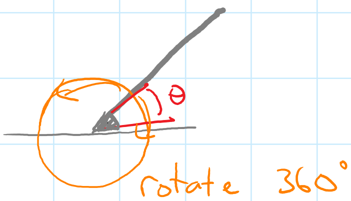
Configuration Spaces of Polygonal Chains

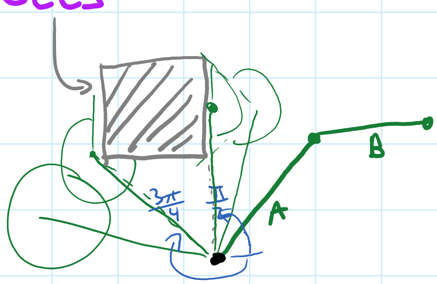
Example: 1-link robotic arm

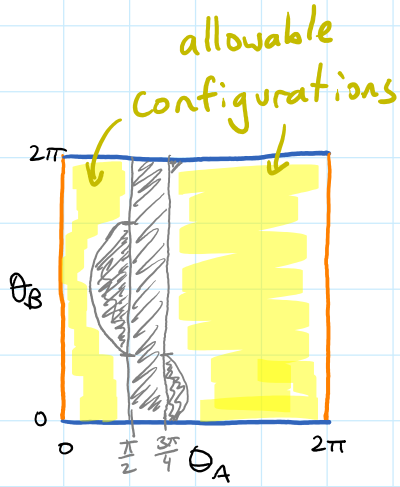


Configuration Space: space of all possible combinations of angles at all joints along the robotic arm

configuration space is a circle, S^1

OBSTACLES:



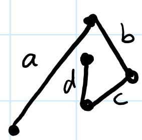


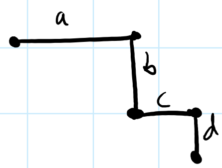
Question: For polygonal chains, is the configuration space connected?

That is, for a given polygonal chain, is it possible to move from one configuration to every other?

Assume segments cannot pass through each other.

2D:



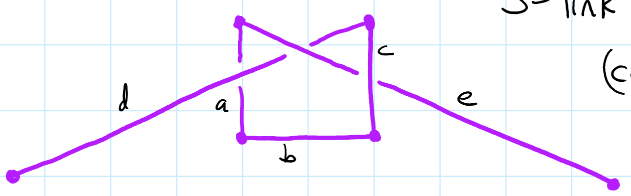


2D config. space is connected

Theorem: Every open polygonal chain in 2D has a motion to a config. in which it is a straight line.
Every closed polygonal chain in 2D has a motion to a convex configuration.

⇨ See Erik Demaine's web site https://erikdemaine.org/linkage/

3D:



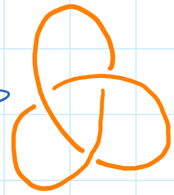
5-link locked chain in 3D
(cannot be straightened)

need: $d > a+b+c$
$e > a+b+c$

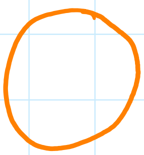
[In 3D, all chains of fewer than 5 links can be unlocked.]

proof relies on the fact that

trefoil knot



and



trivial knot

are different knots.

Open Problems:

- Can a chain in $\mathbb{R}^3$ lock if all its links have the same length?
- Find a polynomial time algorithm to determine whether or not a 3D chain is locked

4D and higher: No polygonal chains are locked.

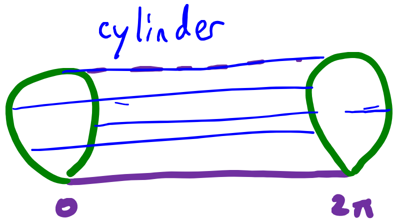
APPLICATION: Protein Folding

A protein can be modeled by a polygonal chain.
Number of links: hundreds to tens of thousands of links

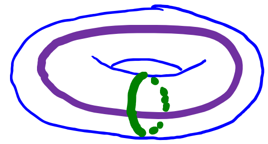
VIDEOS: https://youtu.be/yZ2aY5lxEGE?t=57

https://youtu.be/meNEUTn9Atg

cylinder



identify green circles



Torus!

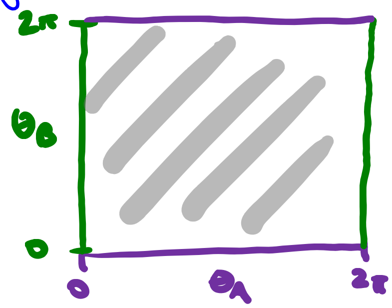
$S^1 \times S^1$ (product of two circles)

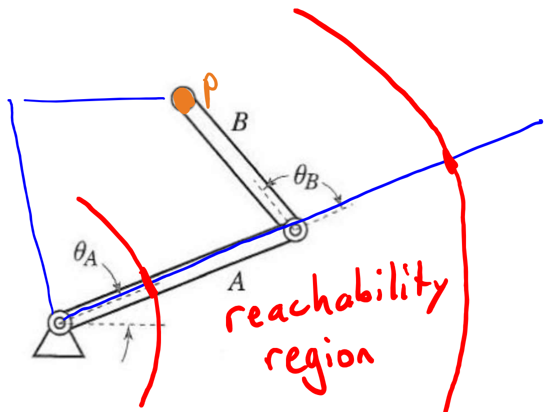
Configuration Spaces

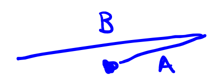
Math 282 Computational Geometry

identify purple segments

1. Consider the following robot arm consisting of two rigid segments, with joints that can each rotate in a full circle.







(a) What is the *configuration space* of the robot arm? (Think in terms of the angles!)

$\{(\theta_A, \theta_B) \mid \theta_A, \theta_B \in [0, 2\pi)\}$

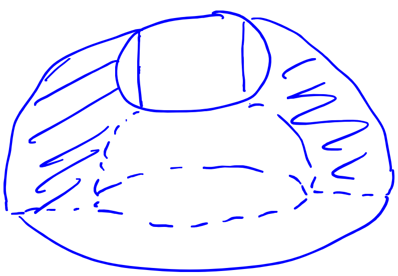
(b) What is the *reachability region* of the robot arm — that is, the set of all possible endpoints of the robot arm? How does this depend on the lengths A and B?

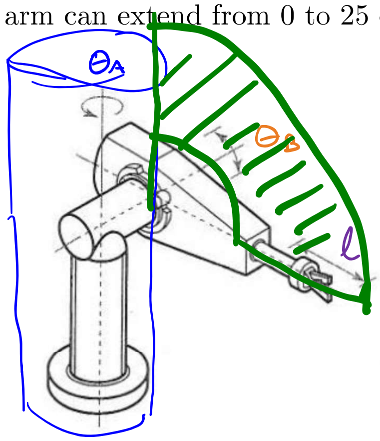
annulus, outer radius $A+B$, inner radius $|A-B|$

(c) Given a point p in the reachability region, how many different configurations of the arm will place the endpoint at p? How can you find these configurations?

2 configurations for any p in the interior of the annulus,
1 config. for p on the boundary

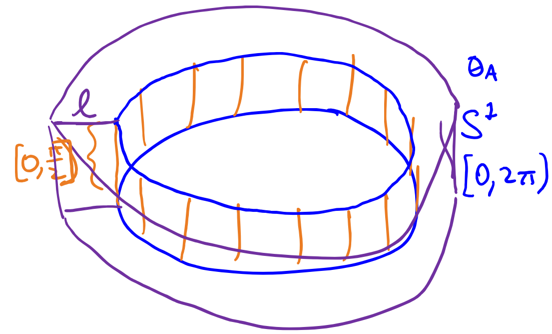
2. Consider the following robot arm. Suppose that the base can rotate a full 360 degrees, the joint can rotate 90 degrees, and the arm can extend from 0 to 25 cm.





(a) What is the *configuration space* of the robot arm?

Solid torus →



(b) What is the *reachability region* of the robot arm?

another solid torus

Images from Adams and Franzosa, *Introduction to Topology, Pure and Applied*.