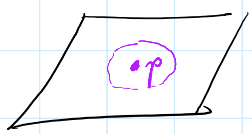
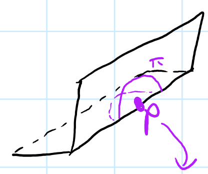
CURVATURE:

The curvature $K(p)$ at a point p on a polyhedron is 2π minus the sum of the face angles at p.

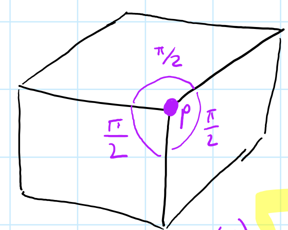
EXAMPLES:



$K(p) = 2\pi - 2\pi = 0$



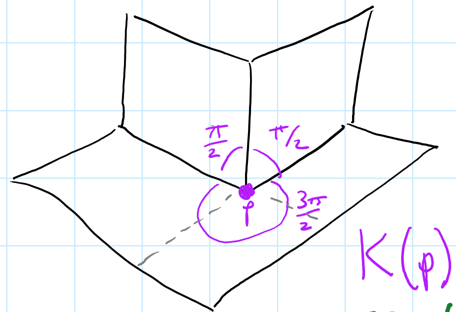
$K(p) = 2\pi - 2\pi = 0$



positive curvature

$K(p) = 2\pi - 3\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$

Curvature is zero except at vertices.

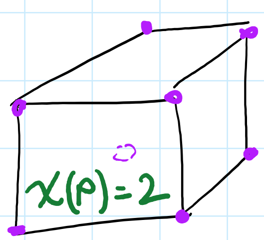


$K(p) = 2\pi - \left(\frac{5\pi}{2}\right) = -\frac{\pi}{2}$

negative curvature

EXERCISE: Sum up curvature over all vertices of:

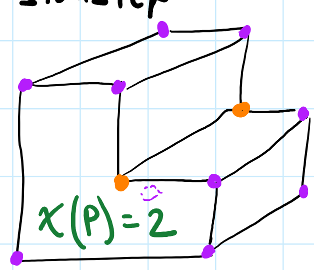
cube



8 vertices, each has curvature $\frac{\pi}{2}$

Sum: $8\left(\frac{\pi}{2}\right) = 4\pi$

stairstep

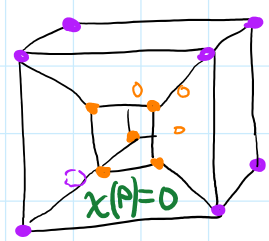


10 vertices with curvature $\frac{\pi}{2}$

2 vertices with curvature $-\frac{\pi}{2}$

Sum: 4π

rectangular "torus"



8 vertices with curvature $\frac{\pi}{2}$

8 vertices with curvature $-\frac{\pi}{2}$

Sum: 0

POLYHEDRAL GAUSS-BONNET THEOREM:

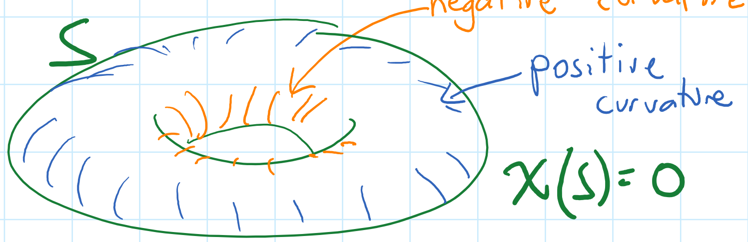
For a polyhedron P, $\sum_{\substack{v \in P \\ \text{vertex}}} K(v) = 2\pi \chi(P)$ ↑ Euler characteristic

GAUSS–BONNET THEOREM:

Let S be a smooth surface without boundary.

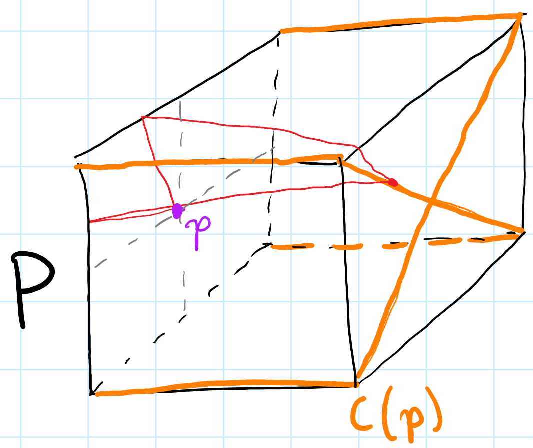
Then: $$\int_S K \, dA = 2\pi \chi(S)$$

↑ Gaussian Curvature (from differential geometry)



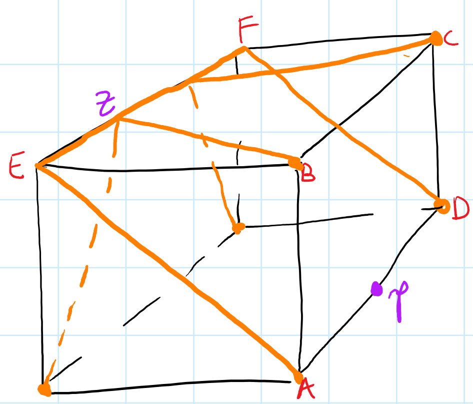
$\chi(S) = 0$

PROBLEM:

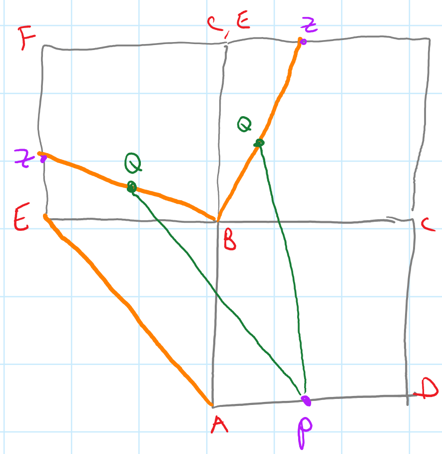


CUT LOCUS:

$C(p)$ is the closure of the set of points on P with more than one shortest path to point p.



F
C
Z
E
B
D
P
A



F
C, E
Z
F
Q
Z
Q
E
B
C
A
P
D