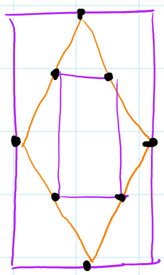
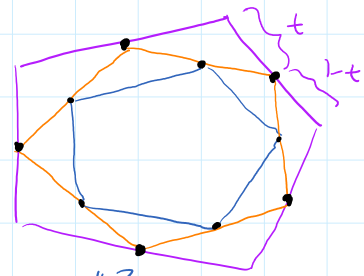
MIDPOINT TRANSFORMATION

What happens if you iterate the transformation?



converges to a regular polygon →

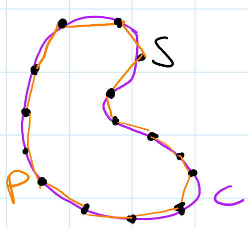
← alternates



"Kasner polygon"?

CURVE RECONSTRUCTION

problem: Suppose we have a finite set of points S sampled from an unknown curve C.
How can we approximate C?



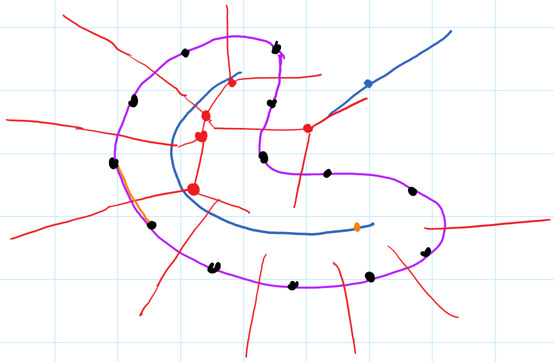
CRUST algorithm:

1. Compute Vor(S). Let V be the set of Voronoi vertices.
2. Compute Delaunay Triangulation Del$(S \cup V)$.
3. Let P be the polygon composed of edges of Del$(S \cup V)$ with both endpoints in S.

Key insights underlying the CRUST algorithm:

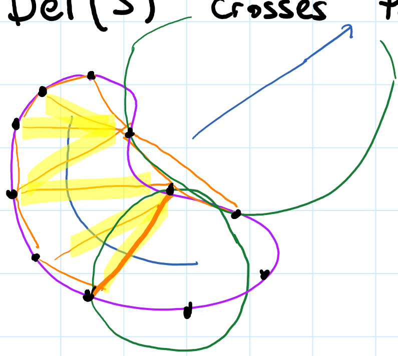
1. Voronoi vertices of Vor(S) lie near the medial axis of C.

   set of points with two closest points on C

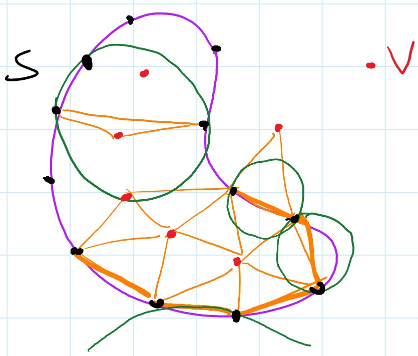


2. Any circumscribing disk of an incorrect edge of Del(S) crosses the medial axis of C.

   an edge between two sample points that are not consecutive on C



3. An incorrect edge of Del(S) cannot also appear in Del(S ∪ V) because ~~any~~ the smallest circumscribing disk contains a vertex of V.

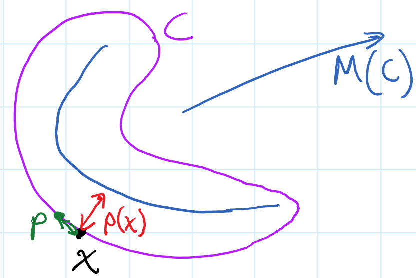


4. Every correct edge of Del(S) also appears in Del(S ∪ V)

If the sample is "dense enough," then any 2 consecutive sample points have a circumscribing disk that does not contain points of V. This implies that the edge connecting the points is in Del(S ∪ V).

Provable Correctness:

Let C be a smooth, closed planar curve.
Let x be a point of C. The local feature size $\rho(x)$ is the shortest distance from x to the medial axis of C.



Let $0 < \varepsilon < 1$. A set S is an ε-sample if each point $x \in C$ has a point $p \in S$ such that $|x - p| \le \varepsilon \rho(x)$

The crust algorithm outputs the correct polygonal reconstruction whenever S is an ε-sample of C with $\varepsilon < \frac{1}{5}$.

Improvements: Tamal Dey — NN-Crust has $\varepsilon < \frac{1}{3}$

others: $\varepsilon < \frac{1}{2}$

Open problem: $\varepsilon \ge \frac{1}{2}$