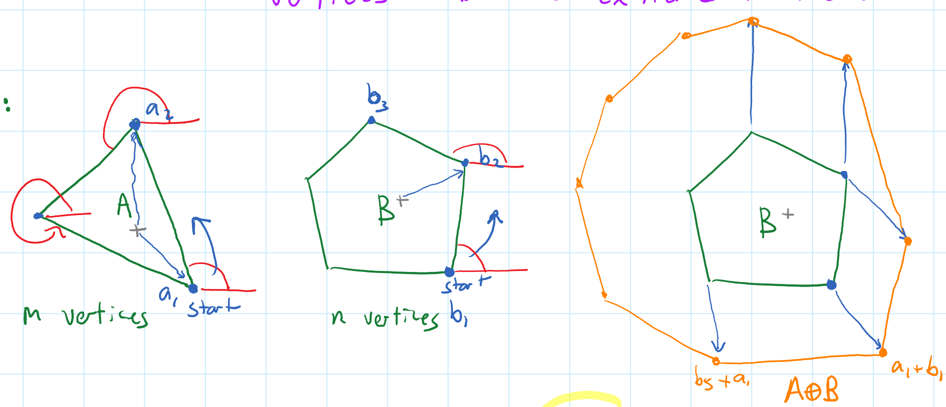
MINKOWSKI SUMS

If A and B are convex polygons with m and n vertices, respectively, then $A \oplus B$ has $\leq m+n$ vertices.

ALGORITHM: Idea: traverse A and B simultaneously, adding vertices that are extreme in the same direction.

example:



$m+n$ vertices, if A and B don't share parallel edges

pseudocode:

minkowskiSum(A, B): # A, B specified by vertices, in CCW order, from the lowest vertex

$m = \text{len}(A)$; $i = 1$

$n = \text{len}(B)$; $j = 1$

while $i \leq m$ or $j \leq n$:

add vertex $a_i + b_j$ to obtain a vertex of $A \oplus B$

$\alpha =$ angle of $\overline{a_i a_{i+1}}$ with positive x-axis

$\beta =$ angle of $\overline{b_j b_{j+1}}$ with positive x-axis

if $\alpha \leq \beta$, then $i{+}{+}$

if $\beta \leq \alpha$, then $j{+}{+}$

complexity: $O(m+n)$

NONCONVEX POLYGONS

case 1: A nonconvex, m vertices

B convex, n vertices

decomposition approach

idea: Triangulate A into $t_1, t_2, \dots, t_{m-2}$

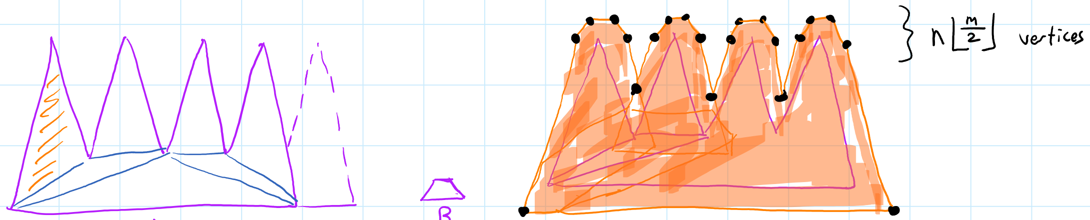
Then: $$A \oplus B = \bigcup_{i=1}^{m-2} (t_i \oplus B)$$

$t_i \oplus B$: $\leq n+3$ vertices

overall $O(mn)$ vertices

number of vertices in $A \oplus B$ is not more than a constant multiple of mn.

example:



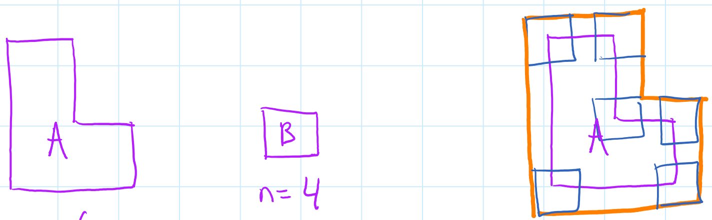
A $m=9$

B $n=4$

$A \oplus B$ 21 vertices

number of spikes: $\lfloor \frac{m}{2} \rfloor$

example:



$m=6$

$n=4$

$A \oplus B$ has 6 vertices

Case 2: both A and B nonconvex

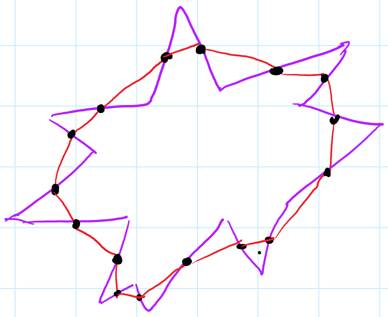
A has m vertices, B has n vertices

Then: $A \oplus B$ has $O(m^2 n^2)$ vertices.

see Figure 5.18 on page 138 in the text.

CURVE SHORTENING

motivation: smooth out sharp corners or noisy data



idea: replace each edge by its midpoint, and connect midpoints of neighboring edges

QUESTIONS:

1. If you apply the midpoint transformation repeatedly, what shape results?

2. Is it possible that the midpoint transformation of a polygon produces something that's not a polygon?