TRIANGULATION ALGORITHMS

TRIANGLE-SPLITTING: First triangulate the hull.
For each interior point, draw edges to the vertices of the triangle that contains it.

COMPLEXITY: For n points:

- Find convex hull: $O(n \log n)$
- Triangulate the hull: $O(h)$ or $O(n)$
  - ↑ number of hull points
- Split interior triangles:
  - Locate triangles containing each interior point
  - For each, draw 3 new edges

  $O(n^2)$; possible $O(n \log n)$ with good data structures

$O(n^2)$ "easy"

$O(n \log n)$ possible

INCREMENTAL ALGORITHM

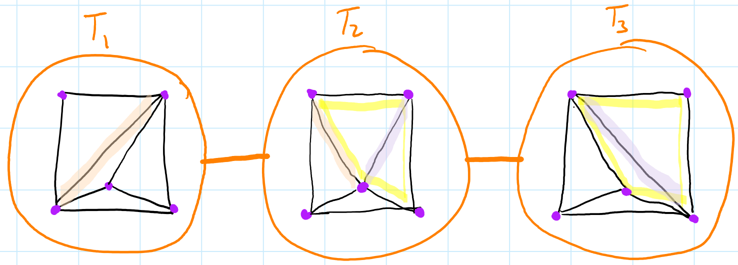
Sort the points. First 3 points make a triangle.
Follow the ordered list of points, adding points and edges to produce a triangulation.

COMPLEXITY:
- Sort the points: $O(n \log n)$
- Add $O(n)$ points, for each, connect to visible points in existing hull. $O(n^2)$
  - visible points ↑ $O(n)$

$O(n^2)$

EDGE FLIPS: Sometimes, two triangulations differ only by an edge flip.

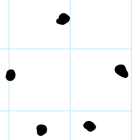
example:

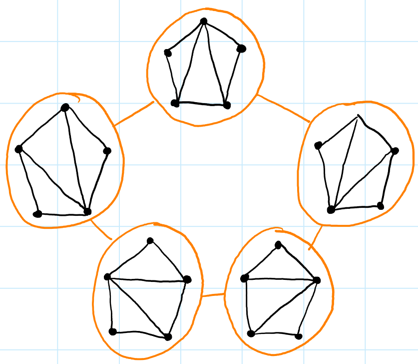


The **FLIP GRAPH** of a point set S is a graph whose nodes are triangulations of S. Two nodes T_1 and T_2 are connected by an edge if one diagonal of T_1 can be flipped to obtain T_2.

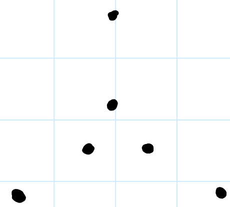
EXERCISES: Draw flip graphs of the following point sets:

(a)

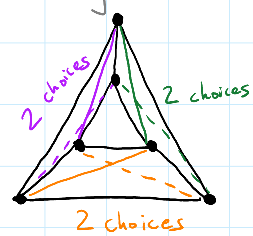


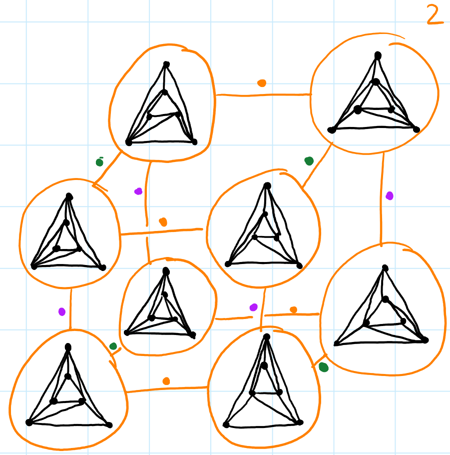


(b)



8 triangulations:





cube!

QUESTION: Is the flip graph of a point set always connected?

Yes, for points in the plane.

Sketch of proof, by induction on n, the number of points:

base case: $n = 3$, then there is only one triangulation

induction: Assume the flip graph is connected for all point sets S with fewer than n points.

Let $S = \{p_1, \ldots, p_n\}$, sorted left-to-right.

Let T_* be the triangulation of S given by the incremental algorithm.

Show that any triangulation T can be made into T_* by edge flips.