Math 262

Section 4.6 Day 36

1. Suppose X_1 and X_2 are iid Unif$[0, 1]$. Let $Y_1 = X_1 + X_2$ and $Y_2 = X_1 - X_2$.

   (a) Find the region of possible values of the pair (Y_1, Y_2).

   (b) Find the inverse transformation functions v_1 and v_2 such that $X_1 = v_1(Y_1, Y_2)$ and $X_2 = v_2(Y_1, Y_2)$.

   (c) Use the tranformation theorem to find the joint pdf of Y_1 and Y_2.

2. Let X_1 and X_2 have joint density $f(x_1, x_2) = \dfrac{1}{x_1^2 x_2^2}$ for $x_1 \geq 1$ and $x_2 \geq 1$. Let $Y_1 = X_1 X_2$ and $Y_2 = \dfrac{X_1}{X_2}$.

   (a) Show that the region of positive joint density for Y_1 and Y_2 is given by $1 \leq Y_1$ and $\frac{1}{Y_1} \leq Y_2 \leq Y_1$.

   (b) Find the joint pdf of Y_1 and Y_2.

3. Let (X, Y) be a random point in the plane, where X and Y are independent standard normal random variables. Let (R, Θ) be the polar coordinates of (X, Y). Find the joint density of R and Θ. Then find the marginal densities of R and Θ. What is the probability that (X, Y) lies inside a circle of radius 1 centered at the origin?