From last class:

4. The number of eggs N found in a nest of a certain species of turtle has a Poisson distribution with mean λ. Each egg has a probability p of being viable, and this event is independent from egg to egg. Find the mean and variance of the number of viable eggs per nest.

rvs: $N \sim \text{Poisson}(\lambda)$ let X = num. of viable eggs $\sim \text{Bin}(N, p)$

mean: $E(X) = E(\underbrace{E(X \mid N)}_{E(X \mid N) = Np}) = E(Np) = p\,E(N) = p\lambda$

variance: $Var(X) = Var(\underbrace{E(X \mid N)}_{Np}) + E(\underbrace{Var(X \mid N)}_{Np(1-p)})$

$= Var(Np) + E(Np(1-p))$

$\boxed{Var(aX+b) = a^2\,Var(X)}$

$= p^2\,Var(N) + p(1-p)\,E(N)$

$= p^2 \lambda + p(1-p)\lambda = \cancel{p^2\lambda} + \cancel{-p^2\lambda} + p\lambda$

$= p\lambda$

Central Limit Theorem

Let $X_1, X_2, \ldots, X_n$ be iid rvs with mean μ and standard deviation σ.

Let $T_n = X_1 + \cdots + X_n$ and $\overline{X}_n = \frac{T_n}{n}$.

Then, as $n \to \infty$:

- The distribution of T_n approaches $N(n\mu, \sigma\sqrt{n})$.
- The distribution of $\overline{X}_n$ approaches $N(\mu, \frac{\sigma}{\sqrt{n}})$.