conditional pmf: $p_{X|Y}(x \mid y) = \dfrac{p(x,y)}{p_Y(y)}$

DISCRETE

conditional pdf: $f_{X|Y}(x \mid y) = \dfrac{f(x,y)}{f_Y(y)}$

CONTINUOUS

Conditional Probability

$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Use the conditional pmf or pdf to compute conditional expectation and conditional variance.

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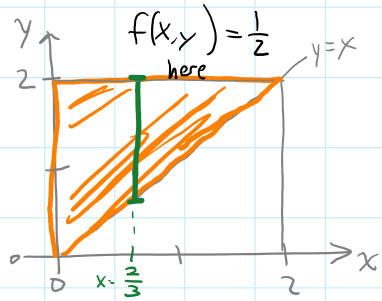
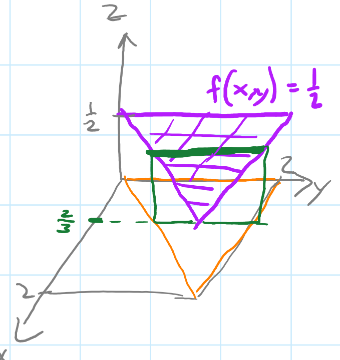
1. $f(x,y) = \frac{1}{2}$ for $0 \le x \le y \le 2$





If $X = \frac{2}{3}$, then $Y \sim \text{Unif}[\frac{2}{3}, 2]$.

So $f_Y(y) = \frac{3}{4}$ for $\frac{2}{3} \le y \le 2$.

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3. $$E(\underbrace{E(X \mid Y)}) = E(X)$$

inside: conditional expectation

outside: expected value of a function of Y

$$E(X \mid Y=y) = \int_{-\infty}^{\infty} x \cdot f_{X|Y}(x \mid y)\, dx = \int_{-\infty}^{\infty} x \cdot \frac{f(x,y)}{f_Y(y)}\, dx$$

function of y

$$E(\underbrace{E(X \mid Y=y)}_{\text{function of } Y}) = \int_{-\infty}^{\infty} E(X \mid Y=y)\, f_Y(y)\, dy$$

$$= \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} x \cdot \frac{f(x,y)}{\cancel{f_Y(y)}}\, dx \right) \cancel{f_Y(y)}\, dy$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x \cdot f(x,y)\, dy\, dx$$

$$= \int_{-\infty}^{\infty} x \underbrace{\boxed{\int_{-\infty}^{\infty} f(x,y)\, dy}}_{f_X(x)}\, dx$$

$$= \int_{-\infty}^{\infty} x \cdot f_X(x)\, dx = E(X)$$