Math 262

Section 4.3 Day 30

1. Let $X \sim \text{Unif}[-1, 1]$ and $Y = X^2$.

   (a) Compute $E(X)$, $E(Y)$, and $E(XY)$. Does $E(XY) = E(X)E(Y)$?

   (b) Are X and Y independent? Why or why not?

Consider the following two scenarios:

I. Two standard, fair dice are rolled. Let X_1 and X_2 be the numbers that appear on the dice.

II. An urn contains balls labeled $1, 2, 3, 4, 5, 6$. Let Y_1 and Y_2 be the numbers on two balls drawn without replacement from the urn.

2. What is the distribution of X_i? How about the distribution of Y_i?

3. What are $E(X_i)$ and $\text{Var}(X_i)$? How about $E(Y_i)$ and $\text{Var}(Y_i)$?

4. What are $E(X_1 + X_2)$ and $\text{Var}(X_1 + X_2)$?

5. What are $E(Y_1 + Y_2)$ and $\mathrm{Var}(Y_1 + Y_2)$?

6. Sketch the pmfs of $X_1 + X_2$ and $Y_1 + Y_2$. How does this help make sense of the means and variances that you found for these sums?

7. Generalize to rolls of n dice: find $E(X_1 + \cdots + E_n)$ and $\mathrm{Var}(X_1 + \cdots + X_n)$.

8. Similarly, generalize to choosing n balls from the urn. Find $E(Y_1 + \cdots + Y_n)$ and $\mathrm{Var}(Y_1 + \cdots + Y_n)$.