1. Suppose that emails arrive in your inbox according to a Poisson process with rate 2 emails per hour. Then the time between successive emails is an exponential random variable with mean 30 minutes.
   (a) What is the probability that an email arrives in the next 20 minutes?
   (b) What is the probability that you don't receive any emails in the next hour?
   (c) What is the standard deviation of the time until the next email?

2. Let $X \sim \text{Exp}(\lambda)$ and $0 < a < b$.
   (a) What is $P(X \geq a)$?
   (b) Show that $P(X > b \mid X > a) = P(X > b - a)$.
   (c) What other distribution satisfies the equality in (b)?
   (d) The property in (b) is special, in the sense that it doesn't hold for most random variables. For example, if $U \sim \text{Unif}[0, 10]$, show that $P(U > 4 \mid U > 3) \neq P(U > 1)$.

3. What is the moment generating function of an exponential random variable?

4. Let $X \sim \text{Exp}(1)$. Find a formula for $E(X^n)$ for positive integers n.

★ **BONUS:** For a positive continuous random variable X with pdf f and cdf F, the *hazard rate* is defined by $h(t) = \frac{f(t)}{1-F(t)}$.

(a) Interpret the hazard rate as a conditional probability. *Hint*: $P(t < X < t + \Delta t) \approx f(t)\Delta t$

(b) Compute the hazard rate for $X \sim \text{Exp}(\lambda)$.