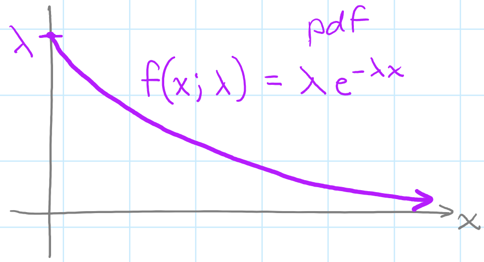
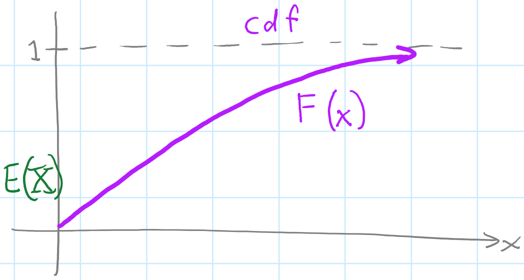
Exponential Distribution

The times between events in a Poisson process are exponentially distributed.



- **pdf:** $f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x} & x > 0 \\ 0 & \text{otherwise.} \end{cases}$

- **cdf:** $F(x) = \begin{cases} 1 - e^{-\lambda x} & \text{for } x > 0 \\ 0 & \text{otherwise.} \end{cases}$



- **mean** $E(X) = \frac{1}{\lambda}$

- **variance:** $Var(X) = \frac{1}{\lambda^2}$ $\Rightarrow \sigma(X) = \frac{1}{\lambda} = E(X)$

- **mgf:** let $X \sim Exp(\lambda)$. Then:

$$M_X(t) = E(e^{tX}) = \int_0^\infty e^{tx} \lambda e^{-\lambda x}\, dx = \lambda \int_0^\infty e^{(t-\lambda)x}\, dx$$

$$= \frac{\lambda}{t-\lambda} e^{(t-\lambda)x} \Big|_{x=0}^{x=\infty} \quad \leftarrow \text{if } t-\lambda<0, \text{ then integral converges}$$

$$= \frac{\lambda}{t-\lambda}(0 - e^0) =$$

$$\boxed{M_X(t) = \frac{\lambda}{\lambda - t} \text{ for } t < \lambda}$$