Math 262

Section 3.2 Day 20

1. Let $U \sim \text{Unif}[0, 5]$.

   (a) What are the mean and variance of U?

   (b) Let $V = 3U + 2$. What are the mean and variance of V?

   (c) What do you think is the distribution of V? *Why?*

2. Let $X \sim \text{Unif}[A, B]$. Use the mgf definition to show that the mgf of X is

$$M_X(t) = \begin{cases} \frac{e^{Bt} - e^{At}}{(B-A)t} & \text{if } t \neq 0, \\ 1 & \text{if } t = 0. \end{cases}$$

   Then use properties of mgfs to verify your answer for 1(c).

3. A stick of length 1 is split at a point U that is uniformly distributed on $(0, 1)$.

(a) What is the expected length of the leftmost piece?

(b) What is the expected length of the longest piece?

(c) What is the expected length of the piece that contains the point p, $0 \leq p \leq 1$?

4. Let X be a random variable that takes on values between 0 and c.

(a) Explain why $E(X^2) \leq cE(X)$.

(b) Use part (a) to show that $\text{Var}(X) \leq c^2[\alpha(1-\alpha)]$, where $\alpha = \dfrac{E(X)}{c}$.

(c) Establish an upper bound on $\alpha(1-\alpha)$ and conclude that $\text{Var}(X) \leq \dfrac{c^2}{4}$.