1. Suppose that $45 \%$ of the phone calls you receive are scam calls. Assume that the probability of a scam call is independent from one call to the next. Let $X$ be the number of calls that you receive until (and including) the next scam call.
(a) What is $P(X=3)$ ?
(b) If $n$ is any positive integer, what is $P(X=n)$ ?
(c) What is $E(X)$ ?
2. Let $Y$ be the number of calls until (and including) the fourth scam call.
(a) What is $P(Y=n)$ ?
(b) What is $E(Y)$ ?
3. An interviewer must find 10 people who agree to be interviewed. Suppose that when asked, a person agrees to be interviewed with probability 0.4.
(a) What is the probability that the interviewer will have to ask exactly 20 people?
(b) What are the mean and standard deviation of the number of people that the interviewer will have to ask?
4. If $X$ has a geometric distribution with parameter $p$, and $k$ is a positive integer, what is $P(X>k)$ ?
5. Scam calls, again. Suppose that $45 \%$ of the phone calls you receive are scam calls.
(a) What is the probability that none of the first 4 calls are scam calls?
(b) If none of the first 4 calls are scam calls, what is the probability that none of the first 7 calls are scam calls?
(c) If none if the first 4 calls are scam calls, what is the probability that none of the first $4+k$ calls are scam calls?
$\star$ BONUS: Show that $\sum_{k=1}^{\infty} k(1-p)^{k-1} p=\frac{1}{p}$. This proves that the mean of a geometric random variable is $\frac{1}{p}$. Hint: Start with a geometric series. What is its sum? Then differentiate.
