NEGATIVE BINOMIAL DISTRIBUTION

An experiment consists of a sequence of independent trials. Each trial results in either "success" or "failure." The probability of success is p for each trial. The experiment stops when a certain number, r, of successes have occurred. Let X be the number of trials necessary to achieve r successes.

Then $X \sim \text{NegativeBinomial}(r, p)$

pmf: $P(X = x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}$

$\binom{x-1}{r-1}$: choose $r-1$ successes among $x-1$ trials

p^r: prob. of r successes

$(1-p)^{x-r}$: prob. of $x-r$ failures

If $r = 1$, then $X \sim \text{Geometric}(p)$

pmf: $P(X = x) = (1-p)^{x-1} p$

For a geometric rv: $X \sim \text{Geometric}(p)$

Geometric tail probability: $P(X > k) = (1-p)^k$

For integers $0 < s < t$:

$$P(X > t \mid X > s) = \frac{P(X > t \text{ and } X > s)}{P(X > s)} = \frac{P(X > t)}{P(X > s)}$$

$$= \frac{(1-p)^t}{(1-p)^s} = (1-p)^{t-s} = P(X > t-s)$$

$$P(X > t \mid X > s) = P(X > t - s)$$

MEMORYLESS PROPERTY of a geometric rv