1. Suppose that during a meteor shower, ten visible meteors per hour are expected.
(a) Let $X$ be the number of visible meteors in one hour. What assumptions must we make in order to say that $X$ has a Poisson distribution?
(b) What is the probability that $5 \leq X \leq 15$ ?
2. Suppose that the number of phone calls an office receives has a Poisson distribution with a mean of 5 calls per hour.
(a) What is the probability that exactly 7 calls are received between 10:00 and 11:00?
(b) What is the probability that more than 7 calls are received between 10:00 and 11:00?
(c) What is the probability that exactly 10 calls are received between $10: 00$ and $12: 00$ ?
3. Suppose that a machine produces items, $2 \%$ of which are defective. Let $X$ be the number of defective items among 500 randomly-selected items produced by the machine.
(a) What is the distribution of $X$ ?
(b) What are the mean and variance of $X$ ?
(c) What is $P(X=12)$ ?
(d) What Poisson distribution approximates the distribution of $X$ ?
(e) Use your Poisson distribution to approximate $P(X=12)$.
4. Let $X \sim \operatorname{Poisson}(\mu)$. Show that $P(X=k)$ increases monotonically and then decreases monotonically as $k$ increases, reaching its maximum when $k$ is the largest integer less than or equal to $\mu$. Hint: Consider $\frac{P(X=k)}{P(X=k-1)}$.
