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POISSON PROCESS: A sequence of discrete occurrences where the average number of occurrences in a fixed time interval is known, but the exact timing of occurrences is random.

EXAMPLES:
- arrival of emails
- phone calls
- emission of radioactive particles
- cars passing a point on a road

POISSON DISTRIBUTION: $X \sim \text{Poisson}(\mu)$ if X counts the occurrences in a Poisson process with mean μ occurrences per time interval.

pmf $$P(X=x) = p(x;\mu) = e^{-\mu}\frac{\mu^x}{x!} \quad \text{for } x = 0, 1, 2, 3, \ldots$$

check: Does it add up to 1?

$$\sum_{x=0}^{\infty} e^{-\mu}\frac{\mu^x}{x!} = e^{-\mu}\sum_{x=0}^{\infty}\frac{\mu^x}{x!} = e^{-\mu}\cdot e^{\mu} = 1$$

Taylor Series $\boxed{e^x = \sum_{k=0}^{\infty}\frac{x^k}{k!}}$

Binomial (n, p) is approximately Poisson (np):

If n is large and p is small, then

$$\underbrace{b(x;n,p)}_{\text{binomial pmf}} \approx \underbrace{p(x;\mu)}_{\text{poisson pmf}} \quad \text{with } \mu = np.$$

Approximation is "good" if $n \geq 100$ and $np \leq 10$.