Math 262

Section 2.3 Day 9

1. Let X be a random variable with pmf given by $p(4) = 0.3$, $p(5) = 0.2$, $p(8) = 0.3$, and $p(10) = 0.2$.

   (a) What is the expected value $E(X)$?

   (b) What is $E(X^2)$?

   (c) What is $\text{Var}(X)$? *Hint: use the shortcut formula!*

   (d) Suppose the random variable is part of a game in which you win $2X - 8$ dollars. Let $Y = 2X - 8$. What is the pmf of Y?

   (e) Use the pmf of Y to find $E(Y)$, your expected winnings in this game.

   (f) Use the pmf of Y to find $E(Y^2)$, and then find is $\text{Var}(Y)$.

   (g) How is $E(Y)$ related to $E(X)$? How is $\text{Var}(Y)$ related to $\text{Var}(X)$?

The next two problems require **Chebyshev's Inequality**: Let X be a discrete random variable with mean μ and standard deviation σ. For any $k \geq 1$,

$$P(|X - \mu| \geq k\sigma) \leq \frac{1}{k^2}.$$

In words, *the probability that X is at least k standard deviations away from its mean is at most $\frac{1}{k^2}$.*

2. Verify that Chebyshev's Inequality holds for the random variable X from Problem 1, using the value $k = 2$. That is, check that $P(|X - \mu| \geq 2\sigma)$ is less than $\frac{1}{(2)^2}$.

3. The number of equipment breakdowns in a manufacturing plant averages 4 per week, with a standard deviation of 0.7 per week.
   (a) Find an interval that includes at least 90% of the weekly figures for the number of breakdowns.
   (b) A plant supervisor promises that the number of breakdowns will rarely exceed 7 in a one-week period. Is the supervisor justified in making this claim? Why?

★ **BONUS:** When flipped, a certain coin comes up heads with probability p. Let X be the number of heads that appear in n flips of this coin.
   (a) What is the probability distribution of X?
   (b) Show that $E(X) = np$.
   Hint: Write $E(X)$ as a sum and factor out np. Then use the binomial theorem to show that the sum equals 1.