BINOMIAL COEFFIENTS – A CLOSER LOOK

combinations $\binom{n}{k}$ — "n choose k", the number of ways of selecting k items from n, without replacement, order unimportant

Consider: $(a+b)^3 = (a+b)(a+b)(a+b) = \binom{3}{3}a^3 + \binom{3}{2}a^2b + \binom{3}{1}ab^2 + \binom{3}{0}b^3$

select a from all 3 factors

select a from 2 of 3 terms

$$(a+b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$$

More generally: $(a+b)^n = \binom{n}{n}a^n + \binom{n}{n-1}a^{n-1}b + \binom{n}{n-2}a^{n-2}b + \cdots + \binom{n}{0}b^n$

They also appear in Pascal's triangle!

key property: $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

$$\begin{array}{ccccccccc}
 & & & & 1 & & & & \\
 & & & 1 & & 1 & & & \\
 & & 1 & & 2 & & 1 & & \\
 & 1 & & 3 & & 3 & & 1 & \leftarrow \\
1 & & 4 & & 6 & & 4 & & 1
\end{array}$$

	ORDER IMPORTANT	ORDER NOT IMPORTANT
WITH REPLACEMENT	n^k	$\binom{k+n-1}{k} = \frac{(k+n-1)!}{(n-1)!\,k!}$
WITHOUT REPLACEMENT	Permutations $\frac{n!}{(n-k)!}$	Combinations $\binom{n}{k} = \frac{n!}{(n-k)!\,k!}$

n: number of possibilities

k: number of items to select