

# BINOMIAL COEFFICIENTS - A CLOSER LOOK

combinations

$\binom{n}{k}$  — "n choose k", the number of ways of selecting k items from n, without replacement, order unimportant

Consider:  $(a+b)^3 = (a+b)(a+b)(a+b) = \binom{3}{3}a^3 + \binom{3}{2}a^2b + \binom{3}{1}ab^2 + \binom{3}{0}b^3$

↑ select a from all 3 factors

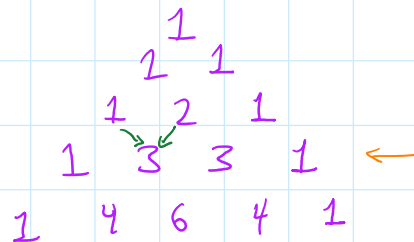
↑ select a from 2 of 3 terms

$$(a+b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$$

More generally:  $(a+b)^n = \binom{n}{n}a^n + \binom{n}{n-1}a^{n-1}b + \binom{n}{n-2}a^{n-2}b^2 + \dots + \binom{n}{0}b^n$

They also appear in Pascal's triangle!

key property:  $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$



	ORDER IMPORTANT	ORDER NOT IMPORTANT
WITH REPLACEMENT	$n^k$	$\binom{k+n-1}{k} = \frac{(k+n-1)!}{(n-1)!k!}$
WITHOUT REPLACEMENT	Permutations $\frac{n!}{(n-k)!}$	Combinations $\binom{n}{k} = \frac{n!}{(n-k)!k!}$

n: number of possibilities

k: number of items to select