Two Discrete Random Variables

For discrete rvs X and Y:

Joint Mass Function: $p(x,y) = P(X=x,\ Y=y)$ ← probability that $X=x$ and $Y=y$ simultaneously

Marginal Mass Functions:

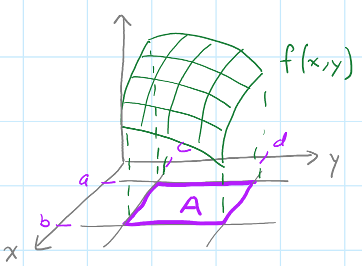
$$\left.\begin{aligned} p_X(x) &= \sum_y p(x,y) \\ p_Y(y) &= \sum_x p(x,y) \end{aligned}\right\} \text{ give probabilities of } X \text{ and } Y \text{ individually}$$

X and Y are **INDEPENDENT** if $p(x,y) = p_X(x)\,p_Y(y)$.

Two Continuous Random Variables

For continuous rvs X and Y:

Joint Density Function: $f(x,y)$ such that for any set A in $\mathbb{R}^2$, $P((X,Y)\in A) = \iint_A f(x,y)\,dA$ — generalizes pdf



$$P(a \le X \le b,\ c \le Y \le b) = P((X,Y)\in A)$$

$$= \int_a^b \int_c^d f(x,y)\,dy\,dx = \iint_A f(x,y)\,dA$$

Marginal Density Functions:

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$$

$$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$$

X and Y are **INDEPENDENT** if $f(x,y) = f_X(x)\,f_Y(y)$.

1(a)

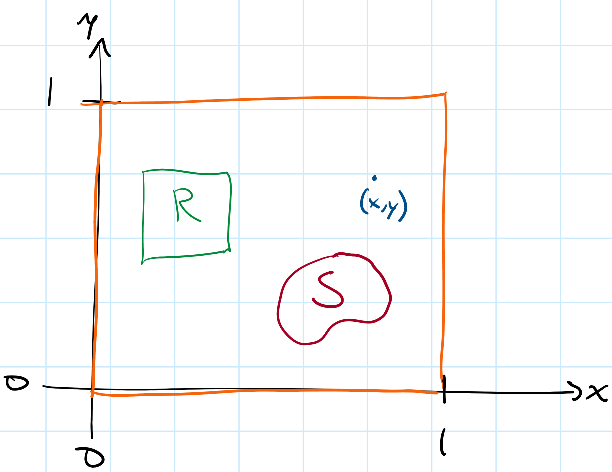
		$X=0$	$X=1$	$X=2$	$X=3$	$p_Y(y)$
Y	0	$\frac{1}{27}$	$\frac{3}{27}$	$\frac{3}{27}$	$\frac{1}{27}$	$\frac{8}{27}$
	1	$\frac{3}{27}$	$\frac{6}{27}$	$\frac{3}{27}$	0	$\frac{12}{27}$
	2	$\frac{3}{27}$	$\frac{3}{27}$	0	0	$\frac{6}{27}$
	3	$\frac{1}{27}$	0	0	0	$\frac{1}{27}$
	$p_X(x)$	$\frac{8}{27}$	$\frac{12}{27}$	$\frac{6}{27}$	$\frac{1}{27}$	

$$p(1,2) = P(X=1, Y=2) = \frac{1}{3}\left(\frac{1}{3}\right)^2 \cdot 3 = \frac{3}{27}$$

probability that 1 student chooses pizza, 2 choose salad bar, 0 choose burger

3 ways of arranging 1 pizza and 2 salad

②



Suppose $\text{Area}(R) = \text{Area}(S)$,

then $P((X,Y) \in R) = P((X,Y) \in S)$

$$\iint_R 1 \, dA = \iint_S 1 \, dA$$

The joint density of X and Y is:

$$f(x,y) = \begin{cases} 1 & \text{if } 0 \le x \le 1,\ 0 \le y \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

(X,Y) is uniformly distributed on the unit square.