There are lots of ways to implement the same simulation, so your solutions may differ from these.

1. Suppose you flip two unfair coins, one of which lands heads with probability 0.4, and the other lands heads with probability 0.6. Estimate the probability that both land heads.

R:

```r
count <- 0               # count starts at zero
for(i in 1:10000){    # loop 10000 times
  # generate random numbers between 0 and 1
  r <- runif(1)
  s <- runif(1)

  # if both coins are heads, then increment counter
  if(r < 0.4 && s < 0.6){
    count <- count + 1
  }
}
print(count) # this is the number of times both coins land heads
```

```r
# more concise code to solve the same problem as above
r <- runif(10000) # 10000 flips of the first coin
s <- runif(10000) # 10000 flips of the second coin
count = (r < 0.4) & (s < 0.6)
#print(count)
print(sum(count))
```

Mathematica:

```
count = 0;
Do[
 r = RandomReal[1];
 s = RandomReal[1];
 If[r < 0.4 && s < 0.6, count += 1],
 10000]
count
```

Exact probability: $(0.4)(0.6) = 0.24$

2. Use simulation to approximate the probability that at least two sixes appear in three rolls of standard, fair dice.

R:

```r
# simulate the probability that at least two sixes appears in
#   three rolls of a standard, fair die
c <- 0
for(i in 1:10000){
  dice <- sample(1:6, 3, TRUE) # three die rolls
  sixes <- sum(dice == 1)        # number of ones in the rolls
  if(sixes >= 2){
    c <- c + 1    # increment counter
  }
}
print(c/10000)
```

Mathematica:

```
count = 0;
Do[
 dice = RandomChoice[Range[6], 3];
 sixes = Count[dice, 6];
 If[sixes ≥ 2, count += 1],
 10000]
N[count / 10000]
```

Exact Probability:

$$P(\text{at least two sixes}) = P(\text{exactly two sixes}) + P(\text{all three sixes})$$

$$= 3\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right) + \left(\frac{1}{6}\right)^3 = \frac{16}{216} \approx 0.074$$

3. Suppose there are 3000 students at St. Olaf College. Estimate the probability that at least 18 students share the same birthday.

R:

```
# simulate the probability that at least 18 people out of 3000 people
#   have the same birthday
count <- 0
for(i in 1:10000){
  bdays <- sample(1:365, 3000, TRUE)
  tab <- table(bdays)
  if(max(tab) >= 18){
    count <- count + 1
  }
}
print(count/10000)
```

Mathematica:

```
count = 0;
Do[
 bdays = RandomChoice[Range[365], 3000];
 tab = BinCounts[bdays];
 If[Max[tab] ≥ 18, count += 1],
 10000]
N[count / 10000]
```

The exact probability is tough to determine, but it's about 0.54.