BINOMIAL COEFFICIENTS: A CLOSER LOOK

We've said $\binom{n}{k}$ is "n choose k", the number of ways of selecting k items from n possibilities without replacement, order not important. COMBINATIONS

Consider: $(a+b)^3 = (a+b)(a+b)(a+b) = 1a^3 + 3a^2b + 3ab^2 + 1b^3$

select either a or b from each factor

$1 = \binom{3}{3}$, $3 = \binom{3}{2}$, $3 = \binom{3}{1}$, $1 = \binom{3}{0}$

More generally: $(a+b)^n = \binom{n}{n}a^n + \binom{n}{n-1}a^{n-1}b + \binom{n}{n-2}a^{n-2}b^2 + \cdots + \binom{n}{2}a^2b^{n-2} + \binom{n}{1}ab^{n-1} + \binom{n}{0}b^n$

$\binom{n}{n} = 1$, $\binom{n}{n-1} = n$, $\binom{n}{1} = n$, $\binom{n}{0} = 1$

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$

Binomial coefficients also appear in Pascal's Triangle

why?

$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \\ & & & & \vdots & & & & \end{array} \longleftrightarrow \begin{array}{ccccccc} & & & \binom{0}{0} & & & \\ & & \binom{1}{0} & & \binom{1}{1} & & \\ & \binom{2}{0} & & \binom{2}{1} & & \binom{2}{2} & \\ \binom{3}{0} & & \binom{3}{1} & & \binom{3}{2} & & \binom{3}{3} \end{array}$$

Selecting k items from n possibilities	order important	order not important
with replacement	1 n^k FCP	4 $\binom{k+n-1}{k}$ balls in boxes
without replacement	2 $\frac{n!}{(n-k)!}$ permutations	3 $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ Combinations