

BINOMIAL COEFFICIENTS: A CLOSER LOOK

We've said $\binom{n}{k}$ is "n choose k", the number of ways of selecting k items from n possibilities without replacement, order not important. } COMBINATIONS

Consider: $(a+b)^3 = (a+b)(a+b)(a+b) = 1a^3 + 3a^2b + 3ab^2 + 1b^3$

$\uparrow \quad \uparrow \quad \uparrow$
 select either a or b from each factor } $\binom{3}{3} \quad \binom{3}{2} \quad \binom{3}{1} \quad \binom{3}{0}$

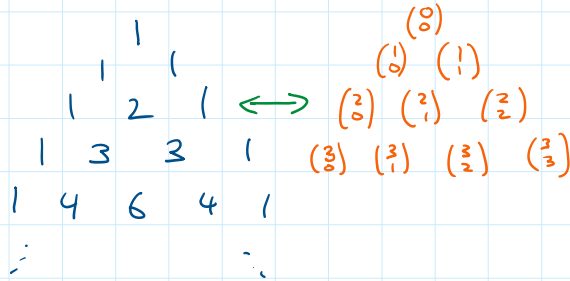
More generally: $(a+b)^n = \binom{n}{n} a^n + \binom{n}{n-1} a^{n-1} b + \binom{n}{n-2} a^{n-2} b^2 + \dots + \binom{n}{2} a^2 b^{n-2} + \binom{n}{1} a b^{n-1} + \binom{n}{0} b^n$

$\uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow$
 $1 \quad n \quad \binom{n}{k} = \frac{n!}{k!(n-k)!} \quad n \quad 1$

Binomial coefficients also appear in Pascal's Triangle

Why?

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$



Selecting k items from n possibilities

	order important	order not important
with replacement	1 n^k <small>FCP</small>	4 $\binom{k+n-1}{k}$ <small>balls in boxes</small>
without replacement	2 $\frac{n!}{(n-k)!}$ <small>permutations</small>	3 $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ <small>Combinations</small>