1. For each of the following experiments, state the sample space and any three events:

(a) A coin is flipped until heads appears, and the number of flips is recorded.

$S = \{1, 2, 3, 4, \ldots\} \cup \{\infty\}$

outcome that heads never appears.

some events: $\{1\}$, $\{2, 4, 6, 8, \ldots\}$, $\{2, 3, 5, 7\}$

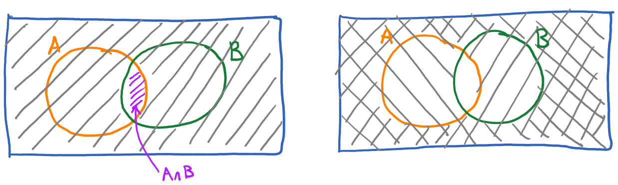
(b) A real number is selected between 0 and 1.

S is the interval $(0, 1)$

some events: $\{\frac{1}{2}\}$, $(0, \frac{1}{2})$, $(\frac{1}{4}, \frac{3}{4})$

2. Let A and B be some events in a sample space. Draw Venn diagrams to illustrate each of the following events:

$(A \cap B)'$ $\qquad$ $A' \cup B'$



How do your diagrams illustrate one of De Morgan's Laws?

$(A \cap B)' = A' \cup B'$ ← the complement of an intersection is a union of complements

3. Write down probability Axiom 3. Let $A_i = \emptyset$ for all $i \in \{1,2,3, \ldots\}$. Explain why this implies $P(\emptyset) = 0$.

If $A_i = \emptyset$ for all $i \in \{1, 2, 3, \ldots\}$, then Axiom 3 says:

$$P(\emptyset \cup \emptyset \cup \emptyset \cup \cdots) = \sum_{i=1}^{\infty} P(\emptyset)$$

Then $P(\emptyset) = \sum_{i=1}^{\infty} P(\emptyset) = P(\emptyset) + P(\emptyset) + P(\emptyset) + \cdots$.

Axiom 1 says $P(\emptyset) \geq 0$. If $P(\emptyset) > 0$, then the equation above is impossible. Therefore, it must be that $P(\emptyset) = 0$.

4. The **Complement Rule** says that for any event A, $P(A) = 1 - P(A')$. (This can be proved using Axiom 3.) Show how the Complement Rule implies that $P(A) \leq 1$ for any event A.

By Axiom 1, $P(A') \geq 0$. Then $-P(A') \leq 0$ and $1 - P(A') \leq 1$.

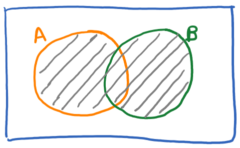
But $P(A) = 1 - P(A')$ by the Complement Rule.

Thus, $P(A) = 1 - P(A') \leq 1$.

$P(A) \leq 1$ is called the NUMERIC BOUND.

5. If A and B are disjoint, Axiom 3 implies that $P(A \cup B) = P(A) + P(B)$. If A and B are *not* disjoint, what is the relationship between $P(A \cup B)$, $P(A \cap B)$, $P(A)$, and $P(B)$?

Venn Diagram:



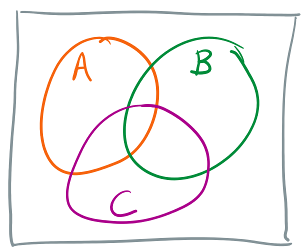
$$\underbrace{P(A \cup B) = P(A) + P(B) - P(A \cap B)}_{\text{ADDITION RULE}}$$

6. Generalize your answer from #5 above to three sets. That is, what can you say about $P(A \cup B \cup C)$?

For any events A, B, C:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

This is the INCLUSION-EXCLUSION PRINCIPLE



BONUS: If $A \subseteq B$ (meaning that A is a subset of B), show that $P(A) \leq P(B)$.

Since $A \subseteq B$, $B = A \cup (A' \cap B)$.

Note that A and $A' \cap B$ are disjoint.

Then $P(B) = P(A) + P(A' \cap B)$

Since $P(A' \cap B) \geq 0$, we conclude that $P(A) \leq P(B)$.