Homework 11

Math 262

due at classtime on Tuesday, December 13

Write your solutions to the following problems clearly and neatly. Make sure to explain your reasoning and provide mathematical details that support your answers. For a few tips on writing solutions, see this helpful guide for mathematical writing.

You may write or type your solutions electronically, or write them on paper and scan or photograph them. Upload a single file containing your solutions to the Homework 11 assignment on Moodle.

Book Problems

- Section 4.4 #69, 77 (page 287)
- Section 4.5 #85, 86, 91, 95, 99 (pages 300–302)
- Section 4.6 #103, 104, 105 (pages 307–309)

Additional Problems

1. Suppose that X is the random variable denoting the number of bacteria per cubic centimeter in water samples and that for a given location, X has a Poisson distribution with mean λ. But λ varies from location to location and has a gamma distribution with parameters α and β. Find expressions for $E(X)$ and $V(X)$ in terms of α and β.

2. Explain in your own words the difference between the Central Limit Theorem and the Law of Large Numbers.

3. The total time X_1 from arrival to completion of service at a fast-food restaurant and the time X_2 spent waiting in line before arriving at the service window have a joint density function given by

$$f(x_1, x_2) = \begin{cases} e^{-x_1} & \text{if } 0 \le x_2 \le x_1 < \infty, \\ 0 & \text{otherwise.} \end{cases}$$

   $Y = X_1 - X_2$ represents the time spent at the service window.

   (a) Find the pdf of Y.

   (b) Find $E(Y)$.

Extra Credit Problem

Two 2-digit numbers and are formed by randomly selecting digits, without replacement, from the digits $1, 2, \ldots, 9$. What is the expected value of the product of the two numbers?