

Math 262

End-of-Semester Review Problems

Day 26

1. Let $\phi(x) = \alpha f(x) + \beta g(x)$. Under what conditions on the constants α and β will $\phi(x)$ be a pdf for all possible pdfs $f(x)$ and $g(x)$?

2. Let $X \sim \text{Exp}(\lambda)$, $0 \leq s$, and $0 \leq t$. Since X is memoryless, is it true that $(X > s + t)$ and $(X > t)$ are independent events?

3. Let X and Y be iid exponential rvs with parameter λ . Let (R, Θ) be the polar coordinates of (X, Y) . What is the joint density of R and Θ ?

4. Suppose B and C are iid $\text{Unif}[0, 1]$. Find the probability that the roots of the equation $x^2 + Bx + C = 0$ are real.

5. Alina makes 100 flips of a fair coin, and Dennis makes 99 flips of a fair coin. What is the probability that Alina gets *more* heads than Dennis?

Hint: Try smaller numbers. Or simulate.

6. X and Y are iid $\text{Unif}[0, 1]$. What is the probability that the closest integer to $\frac{X}{Y}$ is even?

Hint: What is the probability that the closest integer is 0? Or 2? Or 4? Generalize.