Math 262

Section 4.6 Day 24

1. Let X_1 and X_2 be uniformly distributed on the region of the x_1x_2-plane defined by $0 \leq x_1$, $0 \leq x_2$, and $x_1 + x_2 \leq 1$. Let $Y = X_1 + X_2$. Use the following steps to find the density of Y.

   (a) Identify the possible values of Y.

   (b) Sketch the graph $Y = y$ in the x_1x_2-plane.

   (c) Find the region R in the x_1x_2-plane where $Y \leq y$.

   (d) Find the cdf $F_Y(y)$ by integrating the joint density of X_1 and X_2 over the region R.

   (e) Differentiate $F_Y(y)$ to obtain the density $f_Y(y)$.

2. Let X_1 and X_2 have joint density $f(x_1, x_2) = 3x_1$, for $0 \le x_2 \le x_1 \le 1$. Let $Y = X_1 - X_2$. Find the density of Y.

3. The joint density of X_1 and X_2 is $f(x_1, x_2) = 4e^{-2(x_1+x_2)}$ for $X_1 > 0$ and $X_2 > 0$. Find the density of $Y = \frac{X_1}{X_1+X_2}$.

4. Suppose X_1 and X_2 are iid Unif$[0, 1]$. Let $Y_1 = X_1 + X_2$ and $Y_2 = X_1 - X_2$.

   (a) Find the region of possible values of the pair (Y_1, Y_2).

   (b) Find the inverse transformation functions v_1 and v_2 such that $X_1 = v_1(Y_1, Y_2)$ and $X_2 = v_2(Y_1, Y_2)$.

   (c) Use the tranformation theorem to find the joint pdf of Y_1 and Y_2.

5. Let (X, Y) be a random point in the plane, where X and Y are independent standard normal random variables. Let (R, Θ) be the polar coordinates of (X, Y). Find the joint density of R and Θ. Then find the marginal densities of R and Θ. What is the probability that (X, Y) lies inside a circle of radius 1 centered at the origin?