Math 262

Sections 4.3 Day 21

1. Let X and Y be independent uniform variables on $[0, 1]$, and let $W = X + Y$.

   (a) What do you think the pdf of W will look like? Make a guess. Draw a sketch.

   (b) Write down the convolution integral formula for the pdf of W. For what values of x is the integrand nonzero?

   (c) Compute $f_W(w)$ when $0 \leq w \leq 1$.

(d) Compute $f_W(w)$ when $1 \le w \le 2$.

(e) Combine your answers from (c) and (d) to write $f_W(w)$ in piecewise form. Also sketch $f_W(w)$.

2. Use convolution to write an integral that gives the pdf of the sum of three independent Unif$[0, 1]$ random variables. Use Mathematica to evaluate this integral.

Use moment generating functions for the following problems.

mgf reference:

Normal: $e^{\mu t+\sigma^2 t^2/2}$ Exponential: $\frac{\lambda}{\lambda - t}$ Gamma: $\left(\frac{1}{1-\beta t}\right)^{\alpha}$

Geometric: $\frac{pe^t}{1-(1-p)e^t}$ Negative Binomial: $\left(\frac{pe^t}{1-(1-p)e^t}\right)^{r}$

3. Let $X_k \sim N(k, 1)$ for $k \in \{1, 2, \ldots, m\}$, and suppose all of the X_k are independent.

   (a) What is the distribution of $X_1 + X_2 + \cdots + X_m$?

   (b) What is the distribution of $X_1 + 2X_2 + 3X_3 + \cdots + mX_m$?

4. Use moment generating functions to justify the following statements.

   (a) The sum of n independent exponential random variables with common parameter λ has a gamma distribution with parameters $\alpha = n$ and $\beta = 1/\lambda$.

   (b) The sum of n independent geometric random variables with common parameter p has a negative binomial distribution with parameters $r = n$ and p.