

## Math 262

Sections 4.2 and 4.3

Day 20

1. How do  $E(X)$  and  $E(Y)$  relate to  $E(X + Y)$  and  $E(XY)$ ? Does independence play a role?

2. Let  $X \sim \text{Unif}[-1, 1]$  and  $Y = X^2$ .

(a) Compute  $E(X)$ ,  $E(Y)$ , and  $E(XY)$ . Does  $E(XY) = E(X)E(Y)$ ?

(b) Are  $X$  and  $Y$  independent? Why or why not?

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*Consider the following two scenarios:*

**I.** Two standard, fair dice are rolled. Let  $X_1$  and  $X_2$  be the numbers that appear on the dice.

**II.** An urn contains balls labeled 1, 2, 3, 4, 5, 6. Let  $Y_1$  and  $Y_2$  be the numbers on two balls drawn without replacement from the urn.

3. What is the distribution of  $X_i$ ? How about the distribution of  $Y_i$ ?

4. What are  $E(X_i)$  and  $\text{Var}(X_i)$ ? How about  $E(Y_i)$  and  $\text{Var}(Y_i)$ ?

5. What are  $E(X_1 + X_2)$  and  $\text{Var}(X_1 + X_2)$ ?

6. What are  $E(Y_1 + Y_2)$  and  $\text{Var}(Y_1 + Y_2)$ ?

7. Sketch the pmfs of  $X_1 + X_2$  and  $Y_1 + Y_2$ . How does this help make sense of the means and variances that you found for these sums?

8. Generalize to rolls of  $n$  dice: find  $E(X_1 + \cdots + X_n)$  and  $\text{Var}(X_1 + \cdots + X_n)$ .

9. Similarly, generalize to choosing  $n$  balls from the urn. Find  $E(Y_1 + \cdots + Y_n)$  and  $\text{Var}(Y_1 + \cdots + Y_n)$ .