Hypergeometric Distribution

A set contains N items, M of which are "successes" and the rest are "failures." A sample of n items is selected without replacement (each subset of size n is equally likely to be chosen). Let X be the number of successes in the sample.

Then $X \sim \text{Hypergeometric}(n, M, N)$.

choose x out of M successes

choose $n-x$ failures out of $N-M$

pmf: $$P(X=x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$$

total ways of choosing n out of N

mean: $E(X) = n \cdot \frac{M}{N}$ **variance:** $\text{Var}(X) = \frac{N-n}{N-1} \cdot n \cdot \frac{M}{N}\left(1 - \frac{M}{N}\right)$

Negative Binomial Distribution

An experiment consists of a sequence of independent trials. Each trial results in either "success" or "failure." The probability of success is p for each trial. The experiment stops when a certain number, r, of successes have occurred. Let X be the number of trials necessary to achieve r successes.

Then $X \sim \text{NegativeBinomial}(r, p)$. **mean:** $E(X) = \frac{r}{p}$

pmf: $P(X=x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}$ **variance:** $\text{Var}(X) = \frac{r(1-p)}{p^2}$

If $r = 1$, then $X \sim \text{Geometric}(p)$

mean: $E(X) = \frac{1}{p}$

pmf: $P(X = x) = (1-p)^{x-1}p$

variance: $Var(X) = \frac{1-p}{p^2}$