BINOMIAL DISTRIBUTION

$X \sim \mathrm{Bin}(n,p)$ means X is a binomial rv that counts the number of successes in n trials, each with success probability p

pmf: $P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$ for $x = 0, 1, 2, \dots, n$

mean: $E(X) = np$ variance: $Var(X) = np(1-p)$

POISSON DISTRIBUTION

$X \sim \mathrm{Poisson}(\mu)$ if X counts the number of occurrences in a Poisson process with mean μ occurrences per time interval.

pmf: $P(X=x) = e^{-\mu} \dfrac{\mu^x}{x!}$ for $x = 0, 1, 2, 3, \dots$

mean: $E(X) = \mu$ variance: $Var(X) = \mu$

Binomial (n,p) is approximately Poisson (np)

If n is large and p is small, then $b(x;n,p) \approx p(x;\mu)$ with $\mu = np$.

binomial pmf ↑ ↖ Poisson pmf

This approximation is good if $n \geq 100$ and $np \leq 10$.

Historically, the binomial pmf was hard to compute, so the Poisson distribution was useful for approximating binomial probabilities.