Problems from last time (Binomial distribution)

4. A coin that lands on heads with probability p is flipped ten times. Given that a total of 6 heads results, what is the conditional probability that the first three flips are heads, tails, heads (in that order)?

Let $X \sim Bin(10, p)$ be the number of heads in all ten flips.

Let $Y \sim Bin(7, p)$ be the number of heads in the last 7 flips

Then:

$$P(HTH \mid X=6) = \frac{P(HTH \cap X=6)}{P(X=6)} = \frac{P(HTH)\,P(Y=4)}{P(X=6)}$$

$$= \frac{p^2(1-p)\cdot\binom{7}{4}p^4(1-p)^3}{\binom{10}{6}p^6(1-p)^4} = \frac{35}{210} = \boxed{\frac{1}{6}}$$

5. Among persons donating blood to a clinic, 85% have Rh^+ blood. Six people donate blood at the clinic on a particular day.

(a) Find the probability that at most three of the six have Rh^+ blood.

$X \sim Bin(6, 0.85)$ $\qquad$ $P(X \le 3) = B(3; 6, 0.85) = 0.047$

(b) Find the probability that at most one of the six does not have Rh^+ blood.

$$P(X \ge 5) = b(5; 6, 0.85) + b(6; 6, 0.85) = 0.776$$

or: $= 1 - B(4; 6, 0.85)$

(c) What is the probability that the number of Rh^+ donors lies within two standard deviations of the mean number?

$E(X) = 5.1, \qquad \sigma_X = 0.875$

$$P(3.35 < X < 6.85) = P(X=4) + P(X=5) + P(X=6) = 0.953$$

Note: Chebyshev's Inequality implies $P(|X-\mu| < 2\sigma) \ge \frac{3}{4}$, which is true, but the pmf gives a better answer.

(d) The clinic needs six Rh^+ donors on a certain day. How many people must donate blood to have the probability of obtaining blood from at least six Rh^+ donors over 0.95?

Let $Y_n \sim Bin(n, 0.85)$.

We want n such that $P(Y_n \geq 6) \geq 0.95$.

Testing some n, we find:

$$P(Y_8 \geq 6) = 0.895 \quad \text{and} \quad P(Y_9 \geq 6) = 0.966$$

Thus, the clinic needs at least 9 blood donors.

New problems (Poisson distribution)

1. Suppose that during a meteor shower, ten visible meteors per hour are expected.

(a) Let X be the number of visible meteors in one hour. What assumptions must we make in order to say that X has a Poisson distribution?

We must assume that meteors occur independently, and that the average rate over time is known (eg. ten per hour).

(b) What is the probability that $5 \leq X \leq 15$?

If $X \sim \text{Poisson}(10)$, then $P(5 \leq X \leq 15) \approx 0.922$

R: ppois(15, 10) − ppois(4, 10)

2. Suppose that the number of phone calls an office receives has a Poisson distribution with a mean of 5 calls per hour.

(a) What is the probability that exactly 7 calls are received between 10:00 and 11:00?

$X \sim \text{Poisson}(5)$ $\quad P(X = 7) = e^{-5}\frac{5^7}{7!} \approx 0.104$ $\quad$ R: dpois(7, 5)

(b) What is the probability that more than 7 calls are received between 10:00 and 11:00?

$P(X > 7) = 1 - P(X \leq 7) \approx 0.133$ $\quad$ R: 1 − ppois(7, 5)

(c) What is the probability that exactly 10 calls are received between 10:00 and 12:00?

In two hours, the mean number of calls received is ten.

Let $Y \sim \text{Poisson}(10)$. Then $P(Y = 10) = e^{-10}\frac{10^{10}}{10!} \approx 0.125$ $\quad$ R: dpois(10, 10)

3. Suppose that a machine produces items, 2% of which are defective. Let X be the number of defective items among 500 randomly-selected items produced by the machine.

(a) What is the distribution of X? We'll talk about this next time.

(b) What are the mean and variance of X?

(c) What is $P(X = 12)$?

(d) What Poisson distribution approximates the distribution of X?

(e) Use your Poisson distribution to approximate $P(X = 12)$?

4. Let $X \sim \text{Poission}(\mu)$. Show that $P(X = k)$ increaess monotonically and then decreases monotonically as k increases, reaching its maximum when k is the largest integer less than or equal to μ.

This is a fun problem. Next time?