

EXAMPLE: Two standard, fair dice are rolled.

Let X be the sum of the numbers on the dice.

↑ random variable

pmf: probability mass function

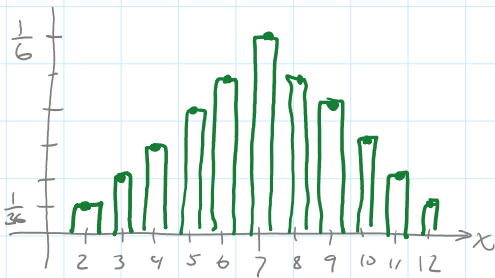
Value x	2	3	4	5	6	7	8	9	10	11	12
$p(x) = P(X=x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

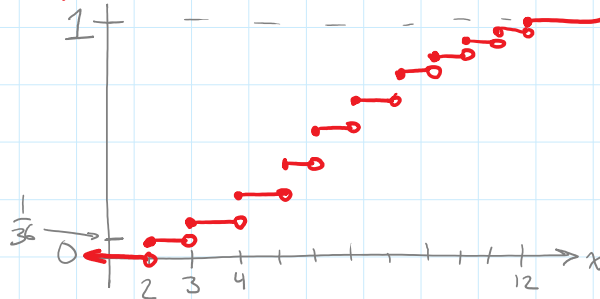
$F(x) = P(X \leq x)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{15}{36}$	$\frac{21}{36}$	$\frac{26}{36}$	$\frac{30}{36}$	$\frac{33}{36}$	$\frac{35}{36}$	1
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cdf: cumulative distribution function

plot the pmf:



plot the cdf:



Expected Value: $E(X) = \sum_x x p(x) = 2 \cdot \frac{1}{36} + 3 \cdot \frac{2}{36} + 4 \cdot \frac{3}{36} + \dots + 11 \cdot \frac{2}{36} + 12 \cdot \frac{1}{36} = 7$

sum of values times their probabilities

Variance: $Var(X) = \sum_x (x - E(X))^2 p(x)$

shortcut formula: $Var(X) = E(X^2) - (E(X))^2$

$Var(X) = \frac{329}{6} - (7)^2 = \frac{35}{6}$

$E(X^2) = \sum_x x^2 p(x) = 2^2 \cdot \frac{1}{36} + 3^2 \cdot \frac{2}{36} + 4^2 \cdot \frac{3}{36} + \dots + 12^2 \cdot \frac{1}{36} = \frac{329}{6}$

↑ squared values ↑ probabilities