Math 262

1. Let X have density given by $f_X(x) = \begin{cases} x+1 & \text{if } -1 \leq x \leq 0, \\ 1-x & \text{if } 0 < x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$

   (a) Sketch the pdf of X.

   (b) Find a formula for the cdf $F_X(x)$. (Also sketch the cdf.)

   (c) Sketch the inverse of $F_X(x)$. Then find a formula for the inverse of $F_X(x)$.

   (d) Write a program to simulate values of X. Simulate thousands of values and make a histogram. Does your histogram look like the density you sketched in part (a)?

2. *Brownian motion* is the random motion of a particle, such as a gas molecule or a tiny piece of dust floating in air.

   We can simulate 1-dimensional Brownian motion with discrete time steps. Suppose that at time 0, a particle starts at position 0. At each time step, the particle moves according to a random variable with distribution given in problem #1. This distribution implies that the particle could move up to one unit left or right in any time step, but it often moves only a tiny distance per time step.

   Specifically, simulate a random variable X_1, which gives the position of the particle at time 1. Simulate another random variable X_2; the position of the particle at time 2 is $X_1 + X_2$. Simulate another random variable X_3; the position of the particle at time 3 is $X_1 + X_2 + X_3$. Continue in this manner to simulate the position of the particle for hundreds of time steps.

   (a) Simulate the Brownian motion described above. Make a plot showing the position of your simulated particle over time.

   (b) Use simulation to answer the question: What is the average number of time steps until the particle's position is at least ten units from the origin?

★ **BONUS:** Simulate 2- or 3-dimensional Brownian motion. Plot the path of your particle. How long does it take for the particle to reach a distance of ten units from the origin? What other questions does this prompt you to ask about Brownian motion?