Math 262

Section 3.4.2 Day 23

1. Suppose that the time between goals in a hockey game is exponentially distributed with mean 18 minutes (ignore timeouts and stoppages). Let X be the time from the start of the game until the second goal occurs.

   (a) Sketch the pdf of X.

   (b) What is the probability that the second goal occurs less than 30 minutes after the game starts?

2. Suppose that a call center receives calls according to a Poisson distribution at a rate of 2 calls per minute. Let Y be the time between 10:00am and the 5^{th} call received after 10:00am.

   (a) Sketch the pdf of Y.

   (b) What are the mean and variance of Y?

   (c) What is $P(Y < 1)$?

3. For large α, the gamma distribution converges to a normal distribution with mean $\alpha\beta$ and variance $\alpha\beta^2$. Investigate this in the case that $\beta = 1$.

   (a) Let $X \sim \text{Gamma}(10, 1)$. Use technology to compute $P(X \leq x)$, for various values of x.

   (b) Let $Z \sim N(10, \sqrt{10})$. Use technology to compute $P(Z \leq x)$ for the same values of x that you used in part (a). Do you find the probabilities to be close to what you found in part (a)?

   (c) Now choose a larger value of α, such as $\alpha = 100$. Compute several probabilities to verify that $X \sim \text{Gamma}(\alpha, 1)$ has nearly the same distribution as $Z \sim N(\alpha, \sqrt{\alpha})$.

4. The *skewness coefficient* of the distribution of random variable X is defined

$$\gamma = \frac{E[(X-\mu)^3]}{\sigma^3}$$

   Discuss with your neighbor: How could you compute the skewness of $X \sim \text{Gamma}(\alpha, \beta)$? Then compute the skewness of X.

★ **BONUS:** Why is $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$? First, use the definition of $\Gamma(\alpha)$ to express $\Gamma\left(\frac{1}{2}\right)$ as an integral. Then make the change of variables $y = \sqrt{2x}$ and relate the resulting expression to the normal distribution.