NORMAL DISTRIBUTION

- Describes the distributions of <u>many</u> physical quantities (e.g. lengths, heights, weights, measurements)
- Related to the Central Limit Theorem (we will study this)

PDF: $X \sim N(\mu, \sigma)$ means the pdf is $f(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$

R FUNCTIONS:

pnorm(x, μ, σ) — computes $P(X \le x)$ if $X \sim N(\mu, \sigma)$

qnorm(p, μ, σ) — computes x such that $P(X \le x) = p$

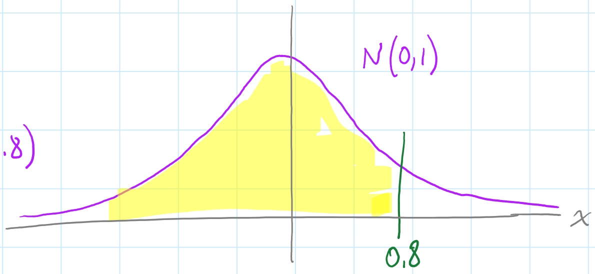
$\mu = 0,\ \sigma = 1$

1. Let Z be a standard normal random variable.

(a) What is $P(Z \le 0.8)$?

R: pnorm(0.8, 0, 1) or pnorm(0.8)
$= 0.788$

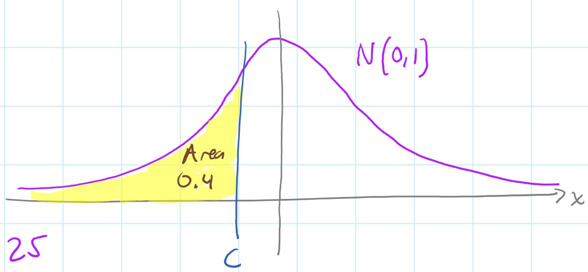
Wolfram Alpha:
cdf[normaldistribution[0,1], 0.8] = 0.788



(b) What number c is such that $P(Z \le c) = 0.4$?

R: qnorm(0.4, 0, 1) or qnorm(0.4)

Wolfram Alpha:
inverse cdf [normaldistribution [0,1], 0.4]
$= -0.25$

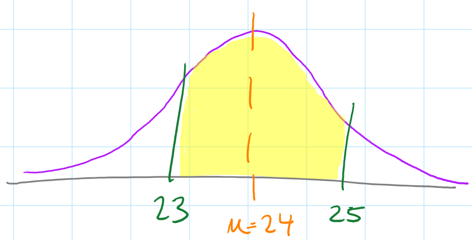


2. Let X be a normal random variable with mean 24 and standard deviation 2.

(a) What is $P(23 \le X \le 25)$?

R: pnorm(25, 24, 2) − pnorm(23, 24, 2)
$= 0.382$

Wolfram Alpha:
cdf[normaldistribution[24,2], 25] − cdf[normaldistribution[24,2], 23]



(b) What number c is such that $P(X \ge c) = 0.2$?

$\longrightarrow P(X \le c) = 0.8$

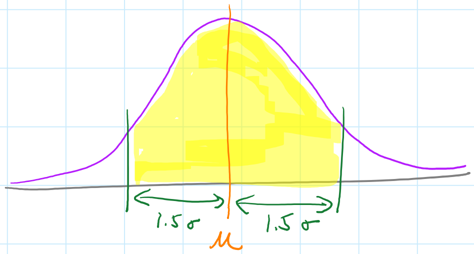
inversecdf[normaldistribution[24,2], 0.8] = 25.68

qnorm(0.8, 24, 2) = 25.68

3. What is the probability that a normal random variable is within 1.5 standard deviations of its mean?

Standard Normal: $P(-1.5 < Z < 1.5) = 0.866$

$= \text{pnorm}(1.5) - \text{pnorm}(-1.5)$



STANDARDIZATION: If $X \sim N(\mu, \sigma)$,

then $Z = \dfrac{X-\mu}{\sigma} \sim N(0,1) \longrightarrow Z\sigma + \mu = X$

Then: $P(\mu - 1.5\sigma < X < \mu + 1.5\sigma) = P(\mu - 1.5\sigma < Z\sigma + \mu < \mu + 1.5\sigma)$

$= P(-1.5\sigma < Z\sigma < 1.5\sigma)$

$= P(-1.5 < Z < 1.5)$

4. Suppose that a fair, 6-sided die is rolled 1000 times. Approximate the probability that the number 6 appears between 150 and 200 times (inclusive). Binomial experiment

Let $X \sim \text{Bin}(1000, \frac{1}{6})$ be the number of 6s rolled.

Then $E(X) = np = \dfrac{1000}{6}$ and $\sigma_X = \sqrt{np(1-p)} = \sqrt{1000\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)} = \sqrt{\dfrac{5000}{36}} \approx 11.8$

Then X is approximately normal with $\mu = \frac{1000}{6}$ and $\sigma = 11.8$.

$\underbrace{P(150 \le X \le 200)}_{\text{this is } 0.926} \approx P(150 \le Y \le 200) = 0.919$

where $Y \sim N\left(\frac{1000}{6}, 11.8\right)$

Normal Approximation to the Binomial

"good" when $np \ge 10$ and $n(1-p) \ge 10$

5. Let $f(x)$ denote the standard normal pdf. Estimate $f(1)$ using only the information in Table A.3 in the text.

to be continued...

6. Let $f(x)$ denote the pdf of the $N(\mu, \sigma)$ distribution. Show that the points of inflection lie at $x = \mu \pm \sigma$. (*Hint*: differentiate twice with respect to x.)