1. How many ways can you distribute 9 cookies to 4 people?

$k=9,\ n=4$ thus: $\binom{9+4-1}{9} = \frac{12!}{9!\,3!} = 220$

2. How many different dominoes can be formed with the numbers 1, 2, …, 6? How about if the numbers 1, 2, …, 12 are used?

→ Chose 2 items from 6, with replacement, order unimportant, in $\binom{2+6-1}{2} = \binom{7}{2} = \frac{7\cdot 6}{2} = 21$ ways

→ For 1,…,12: $\binom{2+12-1}{2} = \binom{13}{2} = \frac{13\cdot 12}{2} = 78$ ways

3. How many ways can 7 identical jobs be assigned to 10 (distinct) people…

(a) …if no person can do multiple jobs?

$\binom{10}{7} = \binom{10}{3} = \frac{10\cdot \cancel{9}^{3}\cdot \cancel{8}^{4}}{\cancel{3}\cdot\cancel{2}\cdot 1} = 120$

(b) …if a single person can do multiple jobs?

(people → boxes, jobs → balls) $\binom{10+7-1}{7} = \binom{16}{7} = \frac{16!}{7!\,9!} = 11440$

4. Seven awards are to be distributed to 10 (distinguishable!) mathletes. How many different distributions are possible if

(a) The awards are identical and nobody gets more than one?

Choose 7 out of 10: $\binom{10}{7} = 120$

(b) The awards are different and nobody gets more than one?

Permutations of 7 selected from 10: $\frac{10!}{3!} = 604{,}800$

(c) The awards are identical and anybody can get any number of awards?

This is selection with replacement, when order doesn't matter, so use the "stars and bars" method: $\binom{7+10-1}{7} = \binom{16}{7} = 11{,}440$

5. Consider the 20 "integer lattice points" (a, b) in the xy-plane given by $0 \leq a \leq 4$ and $0 \leq b \leq 3$, with a and b integers. (Draw a little picture.) Suppose you want to walk along the lattice points from (0,0) to (4,3), and the only legal steps are one unit to the right or one unit up.

(a) How many legal paths are there from (0,0) to (4,3)?

Every legal path involves 7 steps, 3 of which are "up."
Choose any 3 of the 7 steps to be up in $\binom{7}{3} = 35$ ways.

(b) How many legal paths from (0,0) to (4,3) go through the point (2,2)?

Reasoning as before, there are $\binom{4}{2} = 6$ legal paths from (0,0) to (2,2) and $\binom{3}{1} = 3$ legal paths from (2,2) to (4,3).
Thus, there are $6 \cdot 3 = 18$ paths in all.

6. A box contains 5 red, 6 white, and 7 blue balls. The box is stirred and five balls are chosen without replacement. What is the probability that the 5 balls chosen include at least one of each color? Do this in steps:

(a) Let E_1 be the event that *no red ball* is chosen, E_w be the event that *no white ball* is chosen, and E_3 be the event that *no blue ball* is chosen. Find the probabilities $P(E_1)$, $P(E_2)$, and $P(E_3)$.

$$P(E_1) = \frac{\binom{13}{5}}{\binom{18}{5}} \approx 0.150, \quad P(E_2) = \frac{\binom{12}{5}}{\binom{18}{5}} \approx 0.092, \quad P(E_3) = \frac{\binom{11}{5}}{\binom{18}{5}} \approx 0.054$$

(b) Find the probabilities $P(E_1 \cap E_2)$, $P(E_1 \cap E_3)$, $P(E_2 \cap E_3)$, and $P(E_1 \cap E_2 \cap E_3)$.

$$P(E_1 \cap E_2) = \frac{\binom{7}{5}}{\binom{18}{5}} \approx 0.002, \quad P(E_1 \cap E_3) = \frac{\binom{6}{5}}{\binom{18}{5}} \approx 0.0007$$

$$P(E_2 \cap E_3) = \frac{\binom{5}{5}}{\binom{18}{5}} \approx 0.00012$$

Since some balls must be chosen, $P(E_1 \cap E_2 \cap E_3) = 0$

(c) Use inclusion-exclusion to find $P(E_1 \cup E_2 \cup E_3)$.

$$P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3) - P(E_1 \cap E_2) - P(E_1 \cap E_3)$$

$$- P(E_2 \cap E_3) + P(E_1 \cap E_2 \cap E_3)$$

$$\approx 0.150 + 0.092 + 0.054 - 0.002 - 0.0007 - 0.00012$$

$$\approx 0.29$$

(d) Use the preceding result to answer the original question.

We want: $1 - P(E_1 \cup E_2 \cup E_3) \approx 0.71$