If we have equally likely outcomes, then probability reduces to counting.

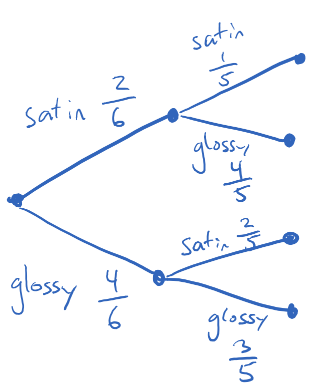
$$\text{probability} = \frac{\text{num. favorable outcomes}}{\text{num. possible outcomes.}}$$

1. A painter has six cans of paint, each containing a different color. Two of the cans contain paint with a satin finish, and four contain glossy paint.

(a) If the painter selects one can of satin paint and one can of glossy paint, how many different color combinations are possible? How does this relate to the Fundamental Counting Principle?

Fundamental Counting Principle: $(2)(4) = 8$ choices

(b) Suppose the painter forgets that the cans contain paint with different finishes, and simply selects two cans at random. Use a tree diagram to help you find the probability that the two selected cans have the same finish.



$\frac{2}{6} \cdot \frac{1}{5} = \frac{2}{30}$

$\frac{2}{6} \cdot \frac{4}{5} = \frac{8}{30}$

$\frac{4}{6} \cdot \frac{2}{5} = \frac{8}{30}$

$\frac{4}{6} \cdot \frac{3}{5} = \frac{12}{30}$

$$P(\text{same finish}) = \frac{2}{30} + \frac{12}{30} = \frac{7}{15}$$

NOTE: **AND** vs. **OR** in counting and probability

AND corresponds to multiplication, as when choosing a glossy paint and a satin paint

OR corresponds to addition, as when choosing either two satin paints or two glossy paints

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COUNTING THE WAYS OF SELECTING k ITEMS FROM n POSSIBILITIES

CASE 1: order important, selection is with replacement

2. Minnesota currently issues license plates that consist of three numbers followed by three letters; for example: 012-ABC. How many different license plates are possible?

Choose the numbers in $10^3 = 1000$ ways, and the letters in 26^3.

By FCP, the total number of plates is $10^3 \cdot 26^3 = 17{,}576{,}000$.

CASE 2: order important, selection is without replacement

permutations of k items out of n possibilities:

$${}_nP_k = P(n,k) = \frac{n!}{(n-k)!}$$

3. How many different 4-letter codes can be made from the letters in the word *PADLOCKS*, if no letter can be chosen more than once? How about 6-letter codes from the letters in *DOGWATCHES*?

"PADLOCKS": $8 \cdot 7 \cdot 6 \cdot 5 = 1680 = \frac{8!}{4!} = \frac{8\cdot7\cdot6\cdot5\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}} = \underbrace{{}_8P_4 = P(8,4)}_{\text{permutation notation}}$

"DOGWATCHES": $10\cdot9\cdot8\cdot7\cdot6\cdot5 = 151{,}200 = \frac{10!}{(10-6)!} = \frac{10!}{4!} = {}_{10}P_4$

CASE 3: order not important, selection without replacement

There are $k!$ orderings of each set of k items.

So, if we have $\frac{k!}{(n-k)!}$ ordered sets of k items chosen from n possibilities,

each unorded set has been counted $k!$ times

Thus, the number of unordered sets is $\frac{n!}{(n-k)!\,k!}$. ← COMBINATION: an unordered selection

4. In a certain lottery, players select six numbers from 1 to 15. For each drawing, balls numbered 1 to 15 are placed in a hopper, and six balls are drawn at random and without replacement. To win, a player's numbers must match those on the balls, in any order.

(a) How many combinations of winning numbers are possible?

$$\binom{15}{6} = \frac{15!}{9!\,6!} = 5005$$

(b) How many combinations would be possible if numbers were chosen between 1 and 24?

$$\binom{24}{6} = \frac{24!}{6!\,18!} = 134{,}596$$

(c) What is the probability that the #1 ball is among the balls chosen? (assume $n = 24$)

$$\text{prob.} = \frac{\text{favorable}}{\text{possible}} = \frac{\binom{23}{5}}{\binom{24}{6}} = \frac{\frac{23!}{18!\,5!}}{\frac{24!}{18!\,6!}} = \frac{23!\,6!}{24!\,5!} = \frac{6}{24} = \frac{1}{4}$$